SOLUTIONS MANUAL
TO ACCOMPANY

KINEMATICS AND DYNAMICS OF MACHINES

SECOND EDITION

GEORGE H. MARTIN

1-1

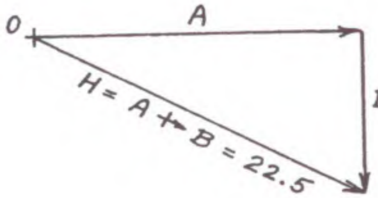
(a) is a mechanism

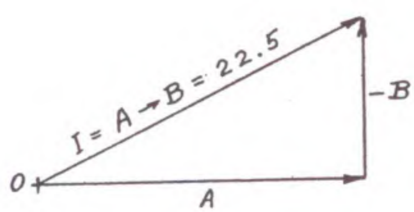
(b) is a structure

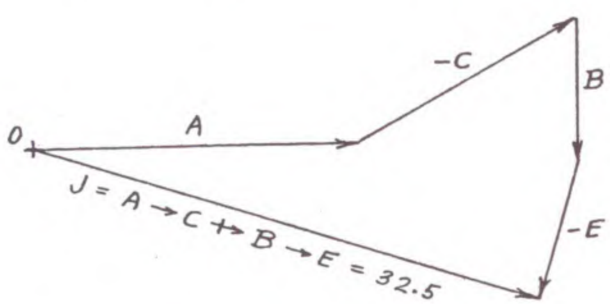
(c) is an unconstrained kinematic chain

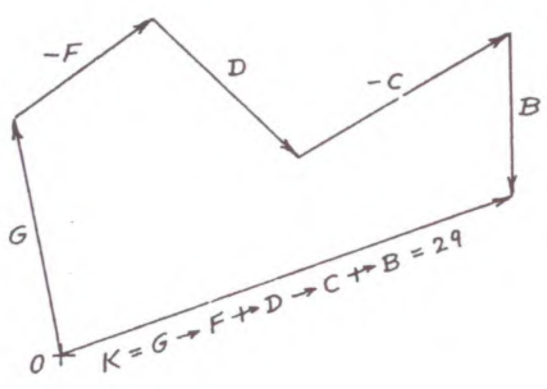
(d) is a mechanism

1-2

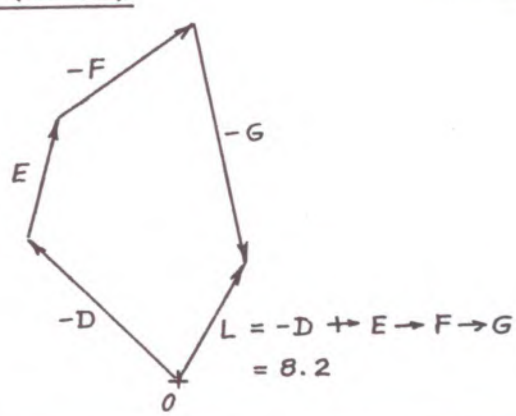








1-2 (CONT.)



1-3

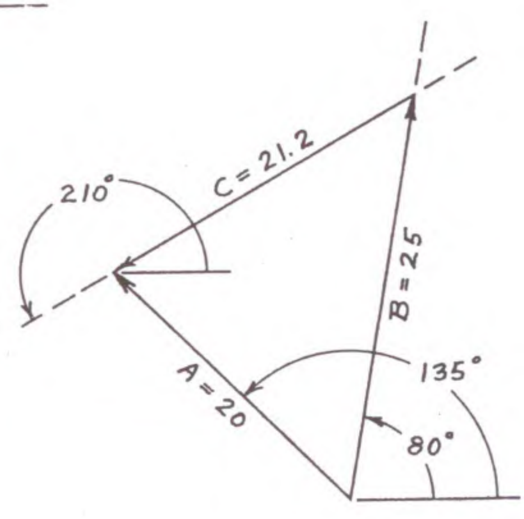
(a) $R = A \mathbin{+\!\!\!\!\to} (-B) \mathbin{+\!\!\!\!\to} C \mathbin{+\!\!\!\!\to} (-D)$
$\quad = A \to B \mathbin{+\!\!\!\!\to} C \to D$

(b) $R = (-A) \mathbin{+\!\!\!\!\to} B \mathbin{+\!\!\!\!\to} C \mathbin{+\!\!\!\!\to} (-D)$
$\quad = (-A) \mathbin{+\!\!\!\!\to} B \mathbin{+\!\!\!\!\to} C \to D$

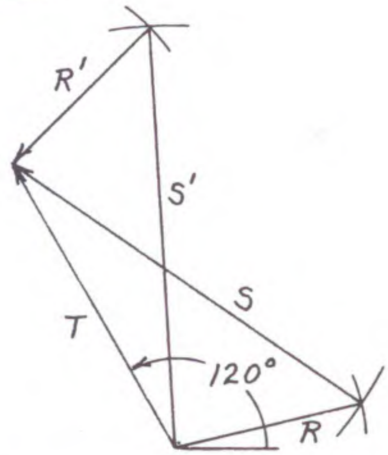
(c) $R = M \mathbin{+\!\!\!\!\to} (-N) \mathbin{+\!\!\!\!\to} (-P)$
$\quad = M \to N \to P$

(d) $R = (-M) \mathbin{+\!\!\!\!\to} (-N) \mathbin{+\!\!\!\!\to} P \mathbin{+\!\!\!\!\to} (-Q)$
$\quad = (-M) \to N \mathbin{+\!\!\!\!\to} P \to Q$

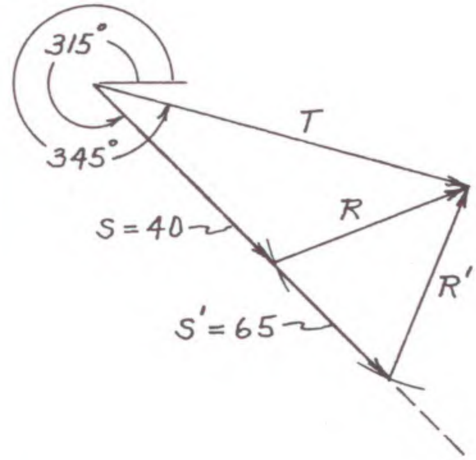
1-4



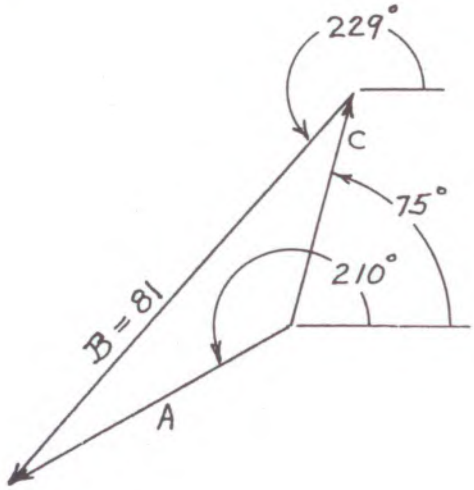
1-5



1-7



1-6



2-1

$V = \pi D n$

$n = \dfrac{V}{\pi D}$

$n = \dfrac{100(12)}{\pi(6)} = \underline{\underline{63.8 \text{ r/min}}}$

2-2

$V_B = R_B \omega \quad , \quad V_c = R_c \omega$

$\dfrac{R_B}{R_c} = \dfrac{V_B}{V_c} = \dfrac{700}{880} = 0.796$

$R_B = 0.796\, R_c$

$R_c - R_B = 2 \text{ in.}, \quad R_c - 0.796 R_c = 2$

$0.204 R_c = 2, \quad R_c = \dfrac{2}{0.204} = \underline{\underline{9.76 \text{ in}}}$

$R_B = 0.796 R_c = 0.796(9.76) = \underline{\underline{7.76 \text{ in}}}$

2-3

a) circumference $= 686\pi = 2\,160 \text{ mm}$

$\dfrac{2160(700)60}{1\,000\,000} = \underline{\underline{90.5 \text{ km/h}}}$

b) $\dfrac{90.5(1000)}{60 \times 60} = \underline{\underline{25.1 \text{ m/s}}}$

c) $\dfrac{700(2\pi)}{60} = \underline{\underline{73.3 \text{ rad/s}}}$

2-4

a) $V = 2\pi R n$

$n = \dfrac{V}{2\pi R} = \dfrac{96\,500(2)}{60(2\pi)0.686} = \underline{\underline{747 \text{ r/min}}}$

b) $747(4) = \underline{\underline{2988 \text{ r/min}}}$

c) $V = 2\pi R n$

$= 2\pi \dfrac{0.0445}{60}(2988) = \underline{\underline{13.9 \text{ m/s}}}$

2-4 (CONT.)

d) $\omega = 2\pi n$

$= 2\pi(2988) = \underline{\underline{18\,774 \text{ rad/min}}}$

e) Piston travel per rev. of engine

$= 2(0.0889) = 0.1778 \text{ m}$

Time for 1 rev. of engine

$= \dfrac{1}{2988} \text{ min}$

$V_{av} = \dfrac{s}{t} = \dfrac{0.1778(2988)}{60}$

$= \underline{\underline{8.85 \text{ m/s}}}$

f) Tire circumference

$= \pi D = \pi(0.686) = 2.16 \text{ m}$

Revs. of wheel per kilometer of car travel

$= \dfrac{1000}{2.16} = 463$

Revs. of engine per kilometer of car travel

$= 4(463) = 1852$

From part (e) piston travel per rev. of engine $= 0.1778 \text{ m}$

$1852(0.1778) = \underline{\underline{329 \text{ m}}}$

2-5

a) $t = \dfrac{s}{V} = \dfrac{0.457}{1.22} = \underline{\underline{0.375 \text{ s}}}$

b) $V = \dfrac{s}{t} = \dfrac{0.457}{0.200} = \underline{\underline{2.29 \text{ m/s}}}$

2-6

$32.2 \text{ km/h} = \dfrac{32\,200}{60 \times 60} = 8.94 \text{ m/s}$

$96.6 \text{ km/h} = \dfrac{96\,600}{60 \times 60} = 26.8 \text{ m/s}$

2-6 (CONT.)

a) $s = V_o t + \frac{1}{2} A t^2$

$$A = \frac{2(s - V_o t)}{t^2} = \frac{2[89.3 - 8.94(5)]}{25} = \frac{2(44.6)}{25} = \underline{\underline{3.57\ m/s^2}}$$

b) $$A = \frac{V - V_o}{\Delta t} = \frac{26.8 - 8.94}{5} = \frac{17.9}{5} = \underline{\underline{3.58\ m/s^2}}$$

2-7

For interval of uniform vel.

$$V = \frac{s}{t} = \frac{5.49}{3} = \underline{\underline{1.83\ m/s}}$$

For interval of constant accel.

$$V = V_o + At$$

$$1.83 = 0 + A(4)$$

$$A = \frac{1.83}{4} = \underline{\underline{0.457\ m/s^2}}$$

2-8

a) $$\omega = \frac{2\pi n}{60} = \frac{2\pi(2000)}{60} = \underline{\underline{209\ rad/s}}$$

$$\alpha = \frac{\omega - \omega_o}{t} = \frac{209 - 0}{5} = \underline{\underline{41.9\ rad/s^2}}$$

b) $$\theta = \frac{1}{2}(\omega_o + \omega)t = \frac{1}{2}(0 + 209)5 = 523\ rad = \frac{523}{2\pi} = \underline{\underline{83.2\ revolutions}}$$

2-9

a) $\omega_o = 1000\ r/min$

$$= \frac{1000(2\pi)}{60} = 105\ rad/s$$

$$\omega = 2000\ rpm = 210\ rad/s$$

$$\alpha = \frac{\omega - \omega_o}{t} = \frac{210 - 105}{20} = \underline{\underline{5.25\ rad/s^2}}$$

b) $$\theta = \frac{1}{2}(\omega_o + \omega)t = \frac{1}{2}(105 + 210)20 = 3150\ rad = \frac{3150}{2\pi} = \underline{\underline{501\ revolutions}}$$

2-10

a) $$\omega = \frac{2\pi n}{60} = \frac{2\pi(2000)}{60} = 209\ rad/s$$

$$\alpha = \frac{\omega - \omega_o}{t} = \frac{209 - 0}{5} = 41.8\ rad/s^2$$

$$R = \frac{3.75}{2(12)} = 0.156\ ft$$

$$A^t = R\alpha = 0.156(41.8) = \underline{\underline{6.55\ ft/s^2}}$$

b) $$A^n = R\omega^2 = 0.156(209)^2 = 0.156(43681) = \underline{\underline{6840\ ft/s^2}}$$

2-11

$V = \dfrac{2\pi R n}{60}$, $R = \dfrac{914}{2} = 457\,mm$

$= \dfrac{2\pi(457)12000}{1000(60)} = \underline{\underline{574\ m/s}}$

$A^n = \dfrac{V^2}{R} = \dfrac{(574)^2}{0.457}$

$= \underline{\underline{7.21 \times 10^5\ m/s^2}}$

2-12

$R = 203\,mm = 0.203\,m$

$\omega = \dfrac{2\pi n}{60} = \dfrac{2\pi(200)}{60}$

$= 20.9\ rad/s$

$\omega t = \theta = 60°$, $\sin 60° = 0.866$

$V = -R\omega \sin\omega t$

$= -0.203(20.9)0.866 = \underline{\underline{-3.67\,m/s}}$

$A = -R\omega^2 \cos\omega t$

$= -0.203(20.9)^2 \cos 60°$

$= -0.203(437)0.5 = \underline{\underline{-44.4\,m/s^2}}$

2-13

a) Time for 1 rev. of crank

$= 2(0.125) = 0.250\,s$

$= \dfrac{0.250}{60} = 0.00416\ min$

$r/min = \dfrac{1}{0.00416} = \underline{\underline{240\ r/min}}$

b) $\omega = \dfrac{2\pi n}{60} = \dfrac{2\pi(240)}{60}$

$= 25.1\ rad/s$

$R = \frac{1}{2}(\text{stroke}) = \frac{356}{2} = 178\ mm$

$V_{max} = R\omega = 0.178(25.1) = \underline{\underline{4.47\,m/s}}$

2-13 (CONT.)

c) $A_{max} = R\omega^2 = 0.178(25.1)^2$

$= 0.178(630) = \underline{\underline{112\ m/s^2}}$

2-14

a) $\omega = 2\pi(7) = 44\ rad/s$

$A_{max} = R\omega^2$

$R = \dfrac{A_{max}}{\omega^2} = \dfrac{0.737}{(44)^2}$

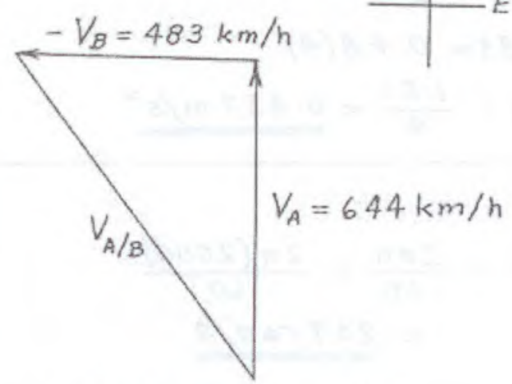
$= \dfrac{0.737}{1936} = \underline{\underline{3.81 \times 10^{-4}\,m}}$

b) $V_{max} = R\omega = 3.81 \times 10^{-4}(44.0)$

$= \underline{\underline{1.68 \times 10^{-2}\,m/s}}$

2-15

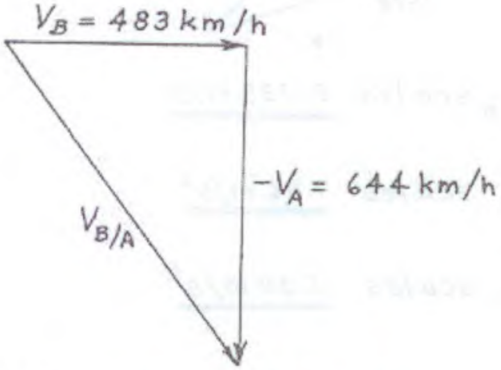
$V_{A/B} = V_A \rightarrow V_B$



$\underline{V_{A/B}\ \text{scales}\ 805\ km/h}$

2-15 (CONT.)

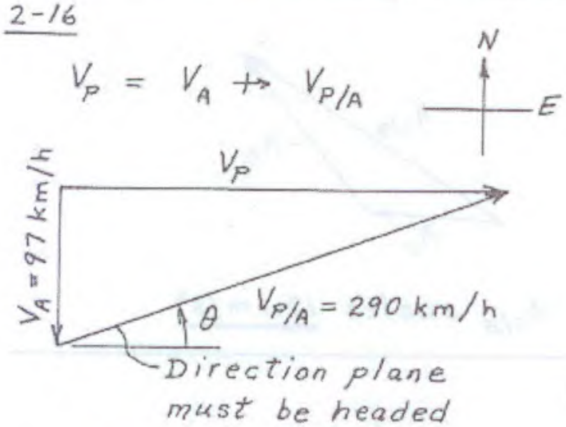
$V_{B/A} = V_B \rightarrow V_A$



$V_{B/A}$ scales $\underline{\underline{805 \text{ km/h}}}$

2-16

$V_P = V_A \leftrightarrow V_{P/A}$



$\underline{\theta \text{ measures } 19.5°}$

V_P scales 274 km/h

$t = \dfrac{s}{V} = \dfrac{644}{274} = \underline{\underline{2.35 \text{ h}}}$

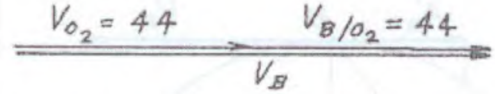
2-17

$V_{O_2} = 30 \text{ mi/h} = \dfrac{30(5280)}{60(60)} = \underline{\underline{44 \text{ ft/s}}}$

$V_{B/O_2} = \underline{\underline{44 \text{ ft/s}}}$

$V_B = V_{O_2} \leftrightarrow V_{B/O_2}$

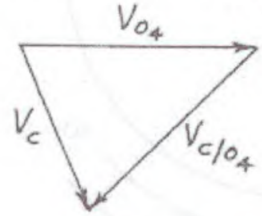
2-17 (CONT.)



$V_B = 44 + 44 = \underline{\underline{88 \text{ ft/s}}}$

$V_{O_4} = 44 \text{ ft/s}, \quad V_{C/O_4} = 44 \text{ ft/s}$

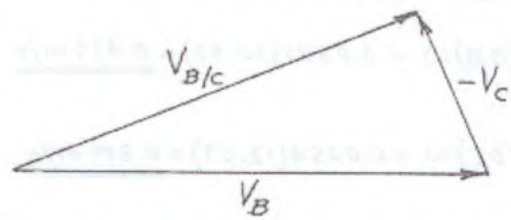
$V_C = V_{O_4} \leftrightarrow V_{C/O_4}$



V_C scales $\underline{\underline{33.9 \text{ ft/s}}}$

$V_{B/C} = V_B \rightarrow V_C$

$= V_B \leftrightarrow (-V_C)$



$V_{B/C}$ scales $\underline{\underline{81 \text{ ft/s}}}$

$\omega_2 = \dfrac{V_{B/O_2}}{O_2B} = \dfrac{44}{1.5} = \underline{\underline{29.3 \text{ rad/s cw}}}$

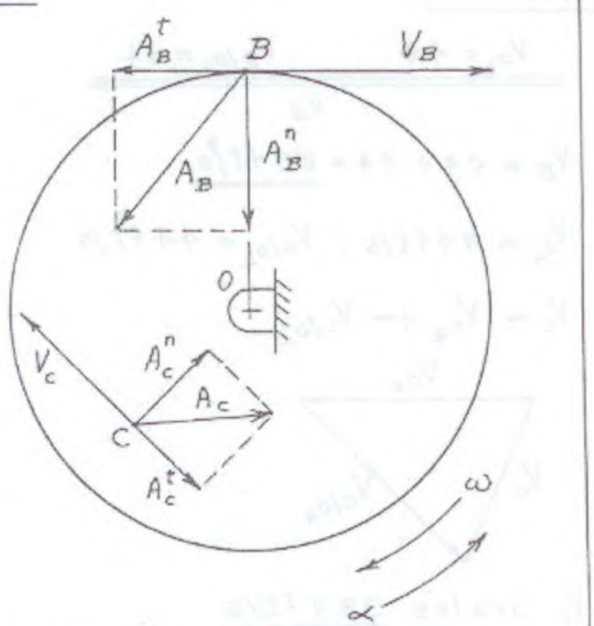
$\omega_4 = \dfrac{V_{C/O_4}}{O_4C} = \dfrac{44}{1} = \underline{\underline{44 \text{ rad/s cw}}}$

$\omega_{2/4} = \omega_2 - \omega_4$

$= -29.3 - (-44)$

$= -29.3 + 44 = \underline{\underline{14.7 \text{ rad/s ccw}}}$

2-18



$\omega = 120\ r/min$

$= \dfrac{120(2\pi)}{60} = 12.57\ rad/s$

$\alpha = 132\ rad/s^2$

$V_B = (OB)\omega = 0.0381(12.57) = \underline{\underline{0.479\ m/s}}$

$V_C = (OC)\omega = 0.0254(12.57) = \underline{\underline{0.319\ m/s}}$

$A_B^n = (OB)\omega^2 = 0.0381(12.57)^2 = \underline{\underline{6.02\ m/s^2}}$

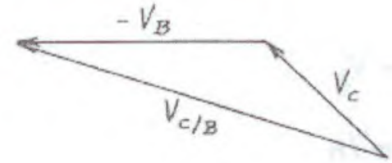
$A_B^t = (OB)\alpha = 0.0381(132) = \underline{\underline{5.03\ m/s^2}}$

$A_C^n = (OC)\omega^2 = 0.0254(12.57)^2 = \underline{\underline{4.01\ m/s^2}}$

$A_C^t = (OC)\alpha = 0.0254(132) = \underline{\underline{3.35\ m/s^2}}$

$V_{C/B} = V_C \rightarrow V_B$

$= V_C +\!\!\!> (-V_B)$



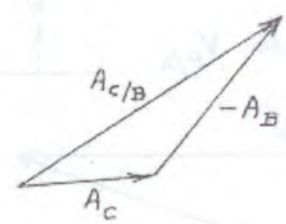
$V_{C/B}$ scales $\underline{\underline{0.735\ m/s}}$

A_B scales $\underline{\underline{7.86\ m/s^2}}$

A_C scales $\underline{\underline{5.30\ m/s^2}}$

$A_{C/B} = A_C \rightarrow A_B$

$= A_C +\!\!\!> (-A_B)$



$A_{C/B}$ scales $\underline{\underline{12.1\ m/s^2}}$

2-19

$\omega = 100\ r/min = \dfrac{100(2\pi)}{60} = 10.47\ rad/s$

$\alpha = 90\ rad/s^2$

$V_B = (OB)\omega = 0.0635(10.47) = \underline{\underline{0.665\ m/s}}$

$V_C = (OC)\omega = 0.0445(10.47) = \underline{\underline{0.465\ m/s}}$

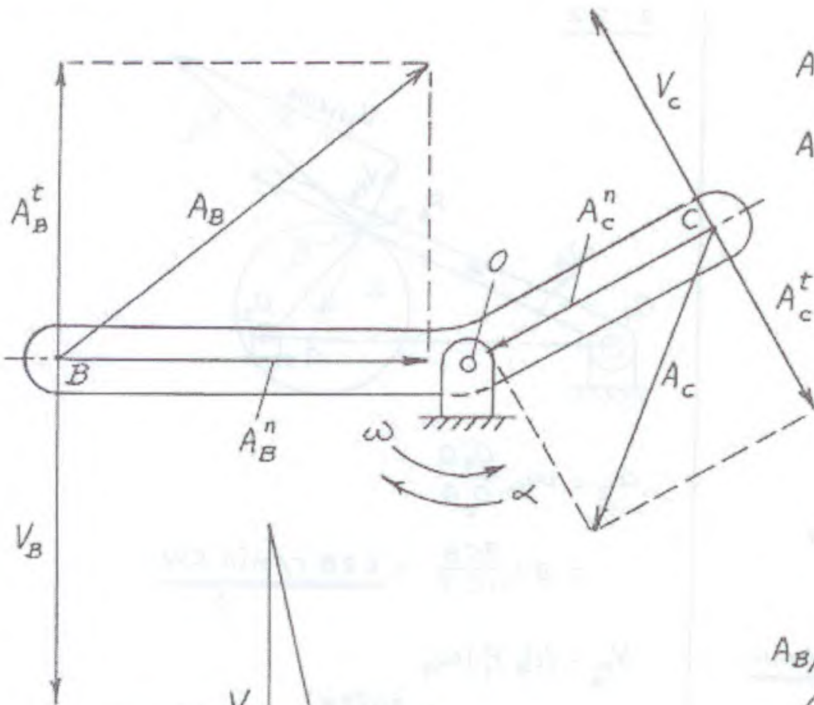
$A_B^n = (OB)\omega^2 = 0.0635(10.47)^2 = \underline{\underline{6.96\ m/s^2}}$

$A_B^t = (OB)\alpha = 0.0635(90) = \underline{\underline{5.72\ m/s^2}}$

$A_C^n = (OC)\omega^2 = 0.0445(10.47)^2 = \underline{\underline{4.87\ m/s^2}}$

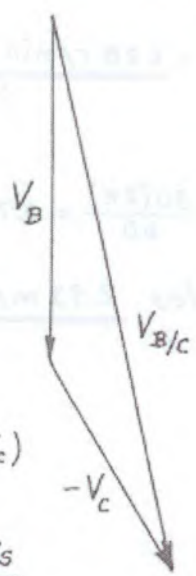
$A_C^t = (OC)\alpha = 0.0445(90) = \underline{\underline{4.00\ m/s^2}}$

2-19 (CONT.)



A_B scales $\underline{8.90 \text{ m/s}^2}$

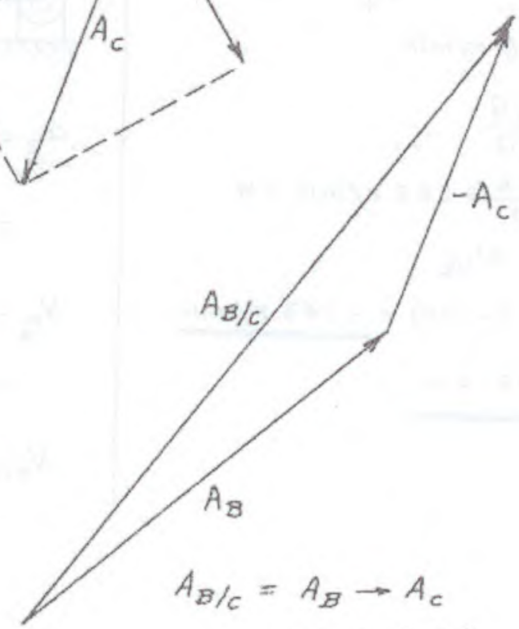
A_c scales $\underline{6.34 \text{ m/s}^2}$



$V_{B/C} = V_B \rightarrow V_C$

$= V_B \leftarrow\!\!+ (-V_c)$

$V_{B/C}$ scales $\underline{1.09 \text{ m/s}}$

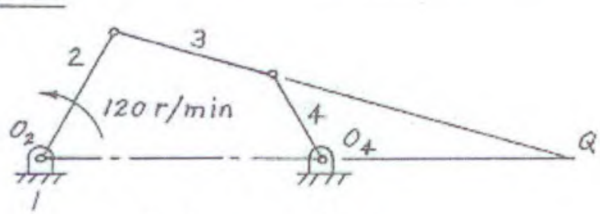


$A_{B/C} = A_B \rightarrow A_c$

$= A_B \leftarrow\!\!+ (-A_c)$

$A_{B/C}$ scales $\underline{14.8 \text{ m/s}^2}$

2-20

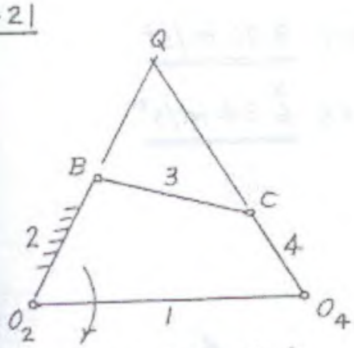


$$\omega_4 = \omega_2 \frac{O_2Q}{O_4Q} = 120 \frac{173}{84}$$

$$= \underline{247 \text{ r/min ccw}}$$

CHAPTER 2. PROPERTIES OF MOTION

2-21



$\omega_{1/2} = 120$ r/min

$$\omega_{3/2} = \omega_{1/2}\frac{O_2Q}{BQ}$$

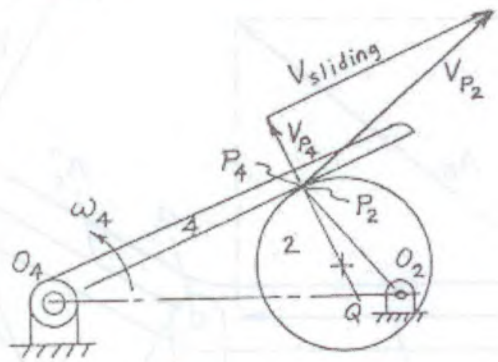
$$= 120\,\frac{87.6}{40} = 263 \text{ r/min cw}$$

$$\omega_{3/1} = \omega_{3/2} - \omega_{1/2}$$

$$= -263 - (-120) = \underline{\underline{-143 \text{ r/min}}}$$

Thus $\underline{\underline{\omega_{3/1}}}$ is cw.

2-22



$$\omega_2 = \omega_4\frac{O_4Q}{O_2Q}$$

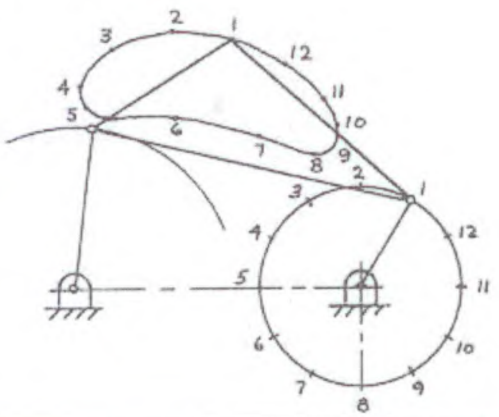
$$= 80\,\frac{95.8}{12.2} = \underline{\underline{628 \text{ r/min cw}}}$$

$$V_{P_4} = (O_4P_4)\,\omega_4$$

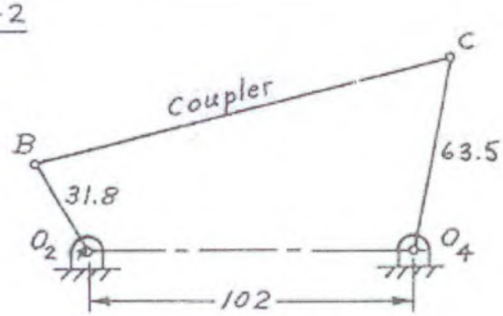
$$= (0.0874)\frac{80(2\pi)}{60} = 0.732 \text{ m/s}$$

$V_{sliding}$ scales $\underline{\underline{2.93 \text{ m/s}}}$

3-1



3-2



1. $31.8 + BC + 63.5 > 102$

   or $BC > 102 - 31.8 - 63.5 > 6.70$ mm

2. $31.8 + 102 + 63.5 > BC$

   or $BC < 197.3$ mm

3. $31.8 + BC - 63.5 < 102$

   or $BC < 102 - 31.8 + 63.5 < 133.7$ mm

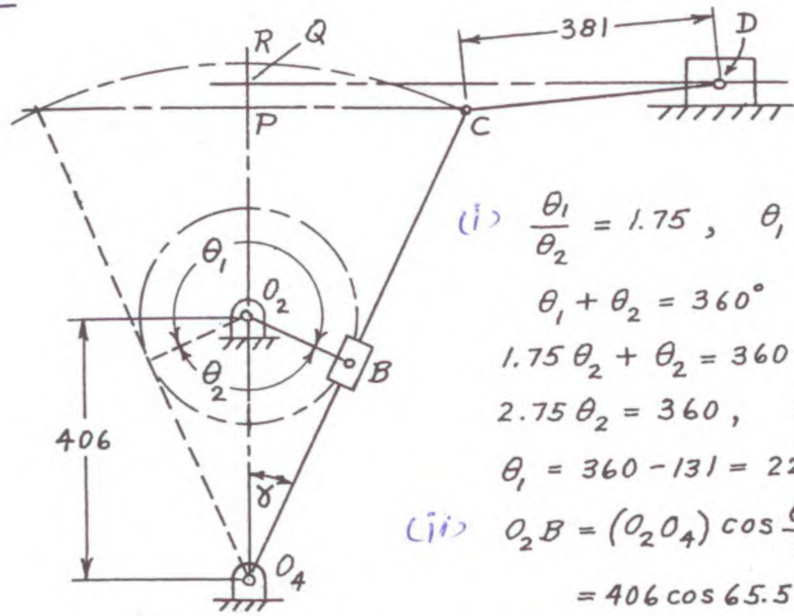
4. $BC - 31.8 + 63.5 > 102$

   or $BC > 102 + 31.8 - 63.5 > 70.3$ mm

$\therefore$ to satisfy all four conditions

$70.3\text{ mm} < BC < 133.7\text{ mm}$

<u>3-3</u>



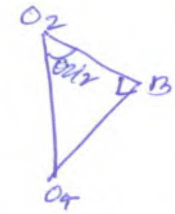
(i) $\dfrac{\theta_1}{\theta_2} = 1.75\,, \quad \theta_1 = 1.75\,\theta_2$

$\theta_1 + \theta_2 = 360°$

$1.75\,\theta_2 + \theta_2 = 360$

$2.75\,\theta_2 = 360\,, \quad \theta_2 = 131°$

$\theta_1 = 360 - 131 = 229°$ $\Delta^{le}\ O_4O_2B$



(ii) $O_2B = (O_2O_4)\cos\dfrac{\theta_2}{2}$

$= 406\cos 65.5° = 406(0.4147) = \underline{168\text{ mm}}$

(iii) $PC = \dfrac{\text{stroke}}{2} = \dfrac{660}{2} = 330\text{ mm}$

$\gamma = 180 - 90 - \dfrac{\theta_2}{2}$

$= 90 - 65.5 = 24.5°$

$\dfrac{PC}{O_4P} = \tan\gamma$

$O_4P = \dfrac{PC}{\tan\gamma} = \dfrac{330}{0.4557} = 724\text{ mm}$

$O_4C = \dfrac{PC}{\sin\gamma} = \dfrac{330}{0.4147} = \underline{\underline{796\text{ mm}}}$

$O_4Q = O_4P + \dfrac{O_4R - O_4P}{2}$

$= 724 + \dfrac{796 - 724}{2}$

$= 724 + 36 = \underline{\underline{760\text{ mm}}}$

(iv) Time for 1 rev. of crank

$= \dfrac{1}{40}\text{ min} = \dfrac{60}{40} = 1.5\text{ s}$

Time for rot. $\theta_1 = \dfrac{229}{360}(1.5) = 0.954\text{ s}$

" " " $\theta_2 = \dfrac{131}{360}(1.5) = 0.546\text{ s}$

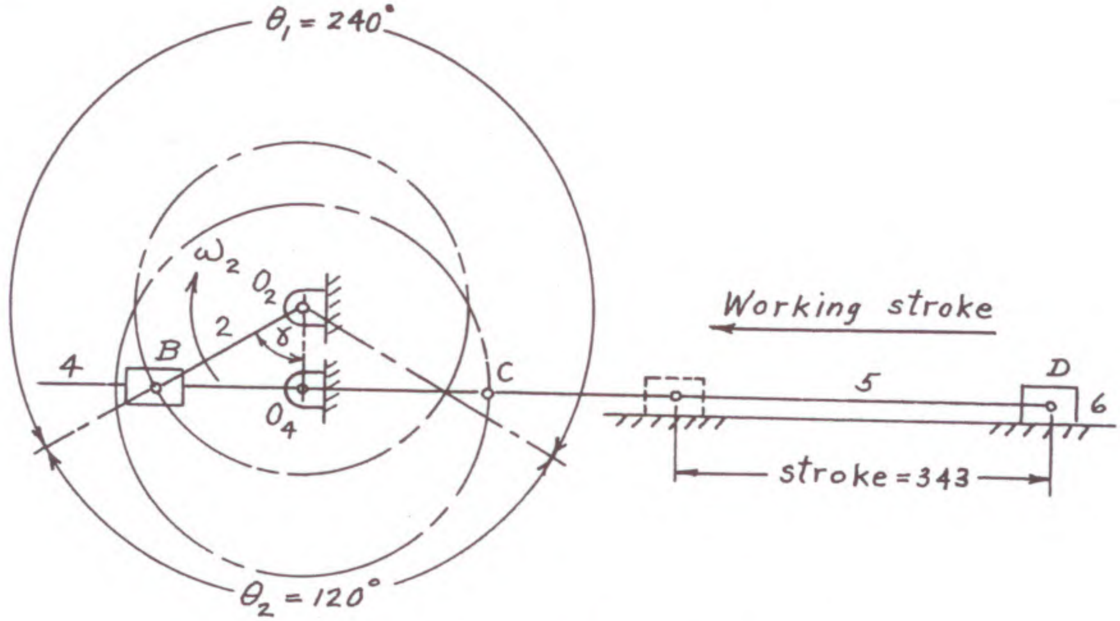
Cutting stroke: → 660 mm → stroke length

$V_{AV} = \dfrac{s}{t} = \dfrac{0.660}{0.954} = \underline{\underline{0.692\text{ m/s}}}$

Return stroke:

$V_{AV} = \dfrac{0.660}{0.546} = \underline{\underline{1.21\text{ m/s}}}$

3-4



$$\frac{\theta_1}{\theta_2} = 2, \quad \theta_1 = 2\theta_2$$

$$\theta_1 + \theta_2 = 360°$$

$$2\theta_2 + \theta_2 = 360°$$

$$3\theta_2 = 360°; \quad \theta_2 = 120°$$

$$\theta_1 = 360 - 120 = 240°$$

$$\gamma = \frac{\theta_2}{2} = \frac{120}{2} = 60°$$

$$O_2B = \frac{O_2O_4}{\cos\gamma} = \frac{76.2}{0.5} = \underline{\underline{152\text{ mm}}}$$

$$O_4C = \frac{\text{stroke}}{2} = \frac{343}{2} = \underline{\underline{172\text{ mm}}}$$

$$CD = 3(O_4C) = 3(172) = \underline{\underline{516\text{ mm}}}$$

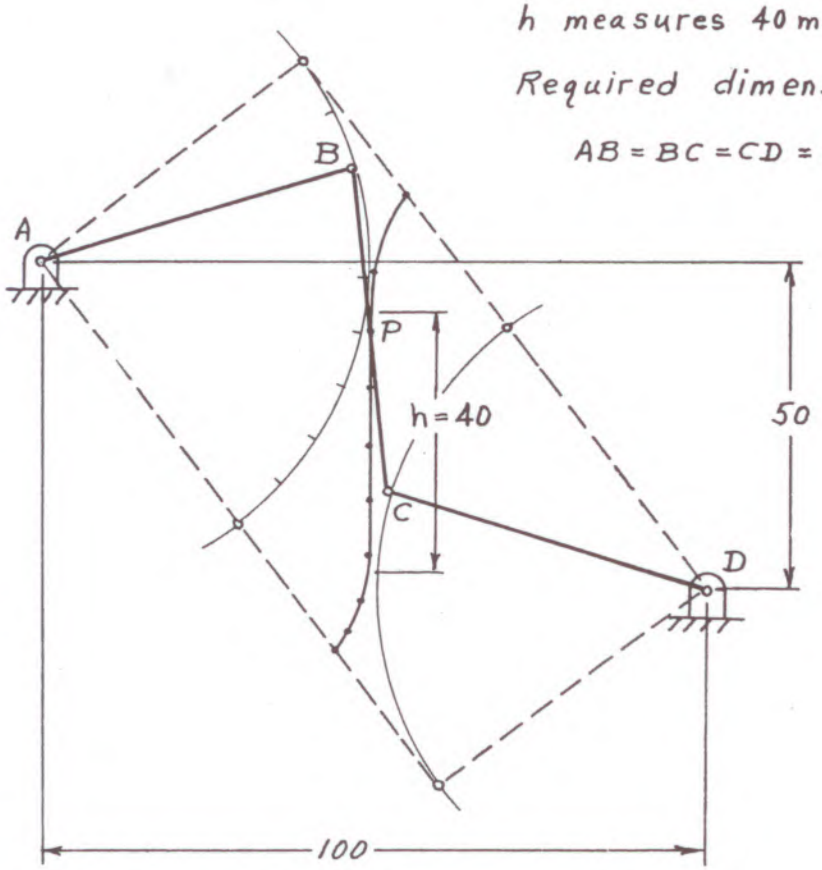
3-5

Assume $AB = BC = CD = 50$ mm

h measures 40 mm approx.

Required dimensions:

$$AB = BC = CD = 50\left(\frac{76}{40}\right) = 95 \text{ mm}$$



3-6 From eq (3-7)

$$\omega_3 = \omega_2 \frac{1-\sin^2\theta_3 \sin^2\delta}{\cos\delta}$$

$\omega_{3\,max.}$ when $\sin\theta_3 = 0$; i.e. $\theta_3 = 0°$

$$\omega_{3\,max.} = \frac{\omega_2}{\cos\delta}$$

$\omega_{3\,min.}$ when $\sin\theta_3 = 1$; i.e. $\theta_3 = 90°$

$$\omega_{3\,min} = \omega_2 \frac{1-\sin^2\delta}{\cos\delta}$$

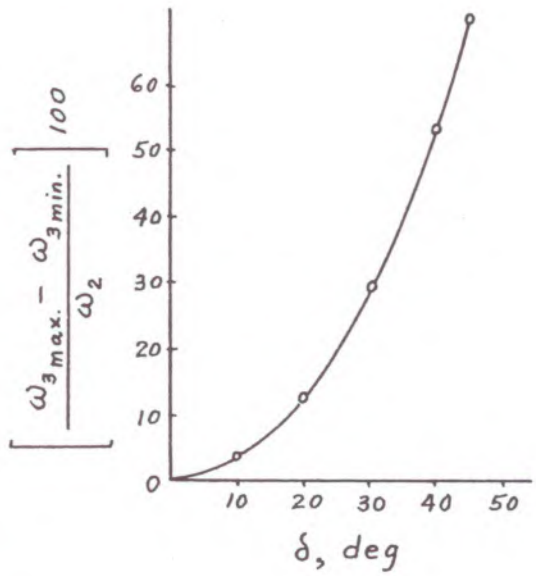
$$\frac{\omega_{3\,max.}-\omega_{3\,min.}}{\omega_2} = \frac{1}{\cos\delta} - \frac{1-\sin^2\delta}{\cos\delta}$$

$$= \frac{1}{\cos\delta}\left[1-(1-\sin^2\delta)\right]$$

$$= \frac{\sin^2\delta}{\cos\delta}$$

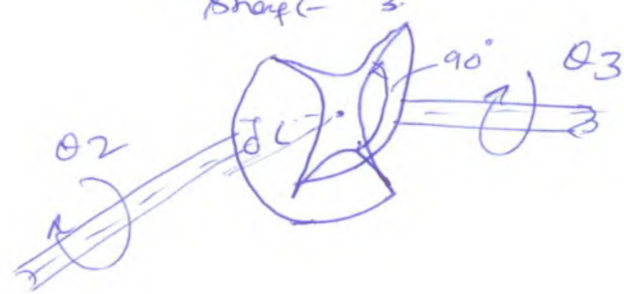
$$= \underline{\underline{\sin\delta \tan\delta}}$$

δ, deg	$\frac{\omega_{3\,max.}-\omega_{3\,min.}}{\omega_2}$
0	0
10	0.0306
20	0.124
30	0.289
40	0.539
45	0.707



$$\frac{\omega_2}{\omega_3} = \frac{\cos\delta}{1-\sin^2\theta_3 \sin^2\delta}$$

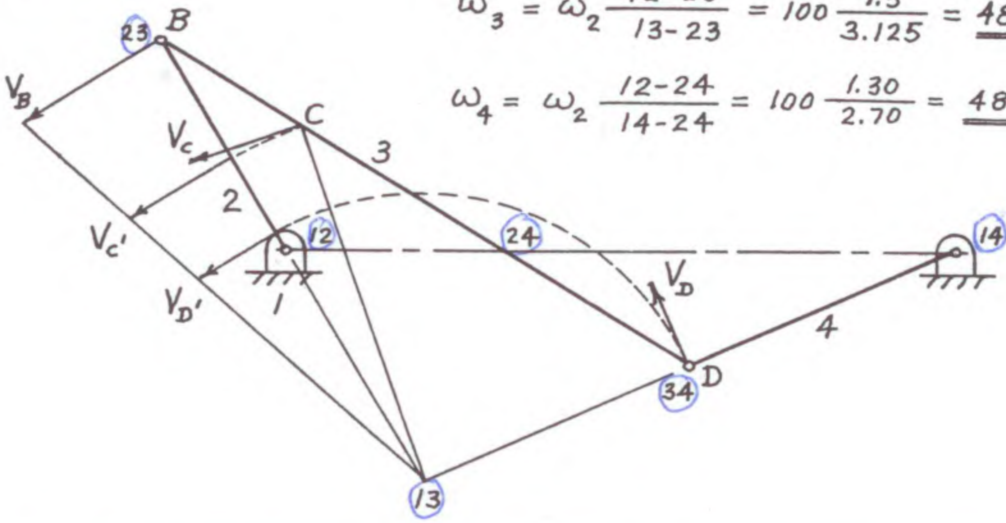
θ_3 - Angle of rotation of Shaft 3



4-1, 5-1, 5-10

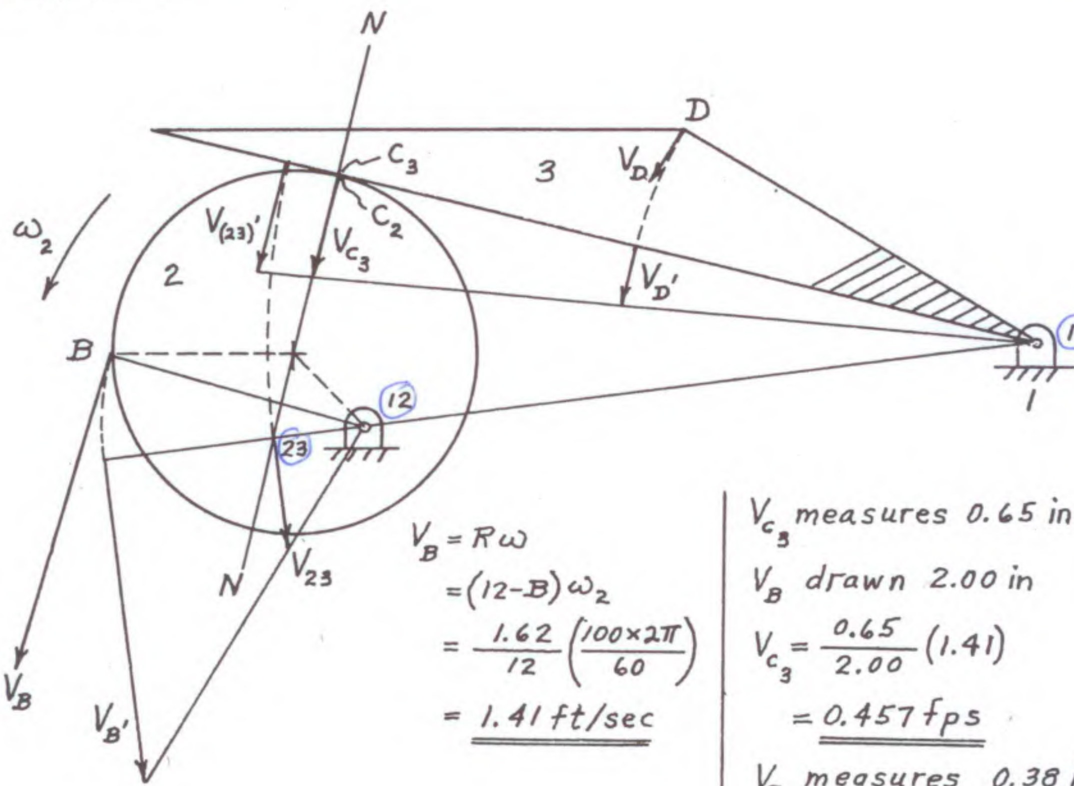
$$\omega_3 = \omega_2 \frac{12-23}{13-23} = 100 \frac{1.5}{3.125} = \underline{\underline{48 \text{ r/min ccw}}}$$

$$\omega_4 = \omega_2 \frac{12-24}{14-24} = 100 \frac{1.30}{2.70} = \underline{\underline{48.1 \text{ r/min cw}}}$$



$\frac{N(N-1)}{2} = \frac{4(4-1)}{2} = \underline{\underline{6}}$

4-2, 5-2, 5-11



$$V_B = R\omega = (12-B)\,\omega_2 = \frac{1.62}{12}\left(\frac{100\times 2\pi}{60}\right) = \underline{\underline{1.41 \text{ ft/sec}}}$$

$$\omega_3 = \omega_2 \frac{12-23}{13-23} = 100 \frac{0.56}{4.64} = \underline{\underline{12.0 \text{ r/min ccw}}}$$

V_{c_3} measures 0.65 in

V_B drawn 2.00 in

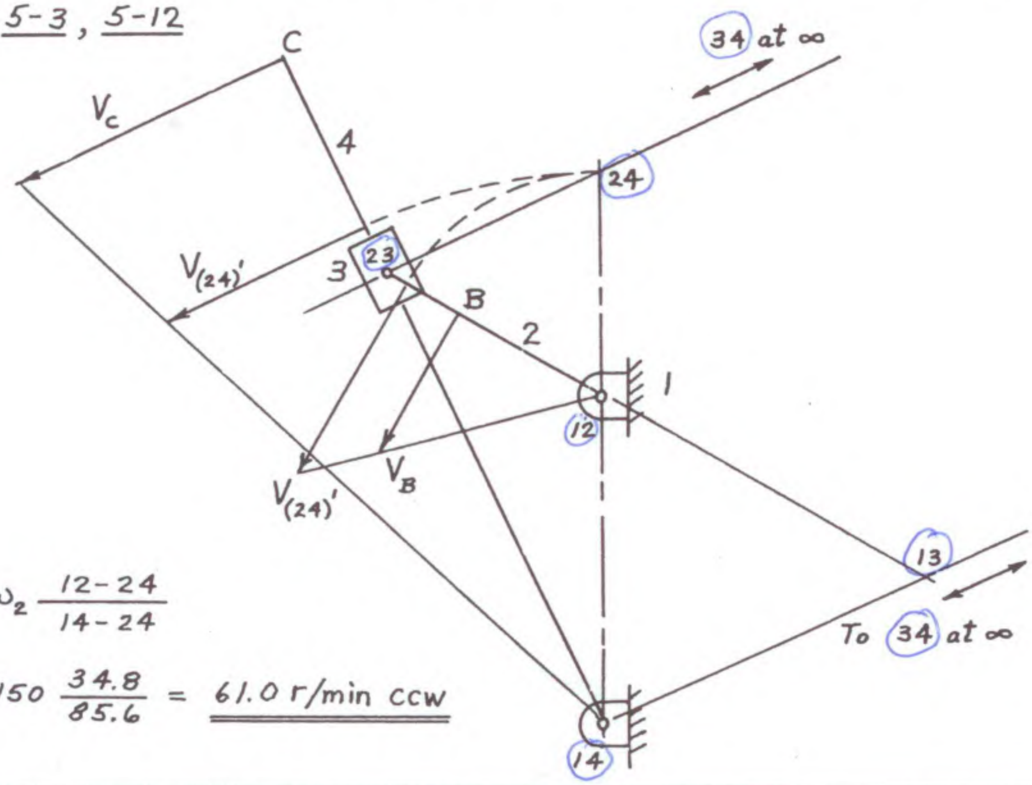
$$V_{c_3} = \frac{0.65}{2.00}(1.41) = \underline{\underline{0.457 \text{ fps}}}$$

V_D measures 0.38 in.

$$V_D = \frac{0.38}{2}(1.41) = \underline{\underline{0.268 \text{ ft/sec}}}$$

$\frac{3(3-1)}{2} = 3$

4-3, 5-3, 5-12



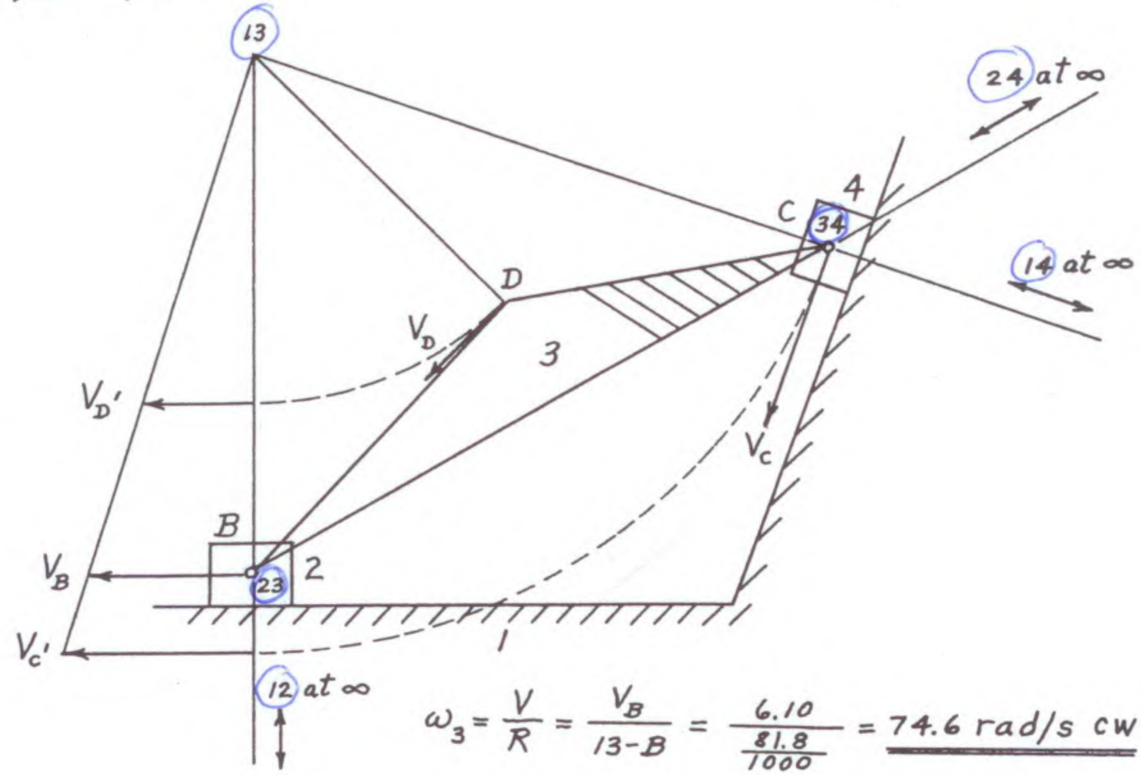
$$\omega_4 = \omega_2 \frac{12-24}{14-24}$$

$$= 150\ \frac{34.8}{85.6} = \underline{\underline{61.0\ \text{r/min ccw}}}$$

$\frac{4(4-1)}{2}$

$= \underline{\underline{6}}$

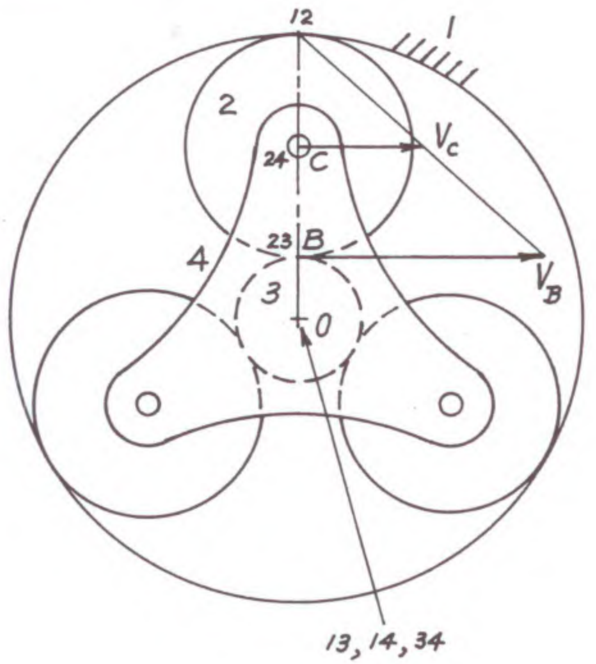
4-4, 5-4, 5-13



$$\omega_3 = \frac{V}{R} = \frac{V_B}{13\text{-}B} = \frac{6.10}{\frac{81.8}{1000}} = \underline{\underline{74.6\ \text{rad/s cw}}}$$

$\frac{4(4-1)}{2} = 6$

4-5, 5-5, 5-14



$$\omega_3 = \frac{V_B}{OB}, \quad \omega_4 = \frac{V_c}{OC}$$

$$\frac{\omega_3}{\omega_4} = \frac{V_B\,(OC)}{(OB)\,V_c}$$

But $V_c = \dfrac{V_B}{2}$

$$\frac{\omega_3}{\omega_4} = \frac{\cancel{V_B}(OC)\,2}{(OB)\cancel{V_B}}$$

Also $OB = 28.6$ mm

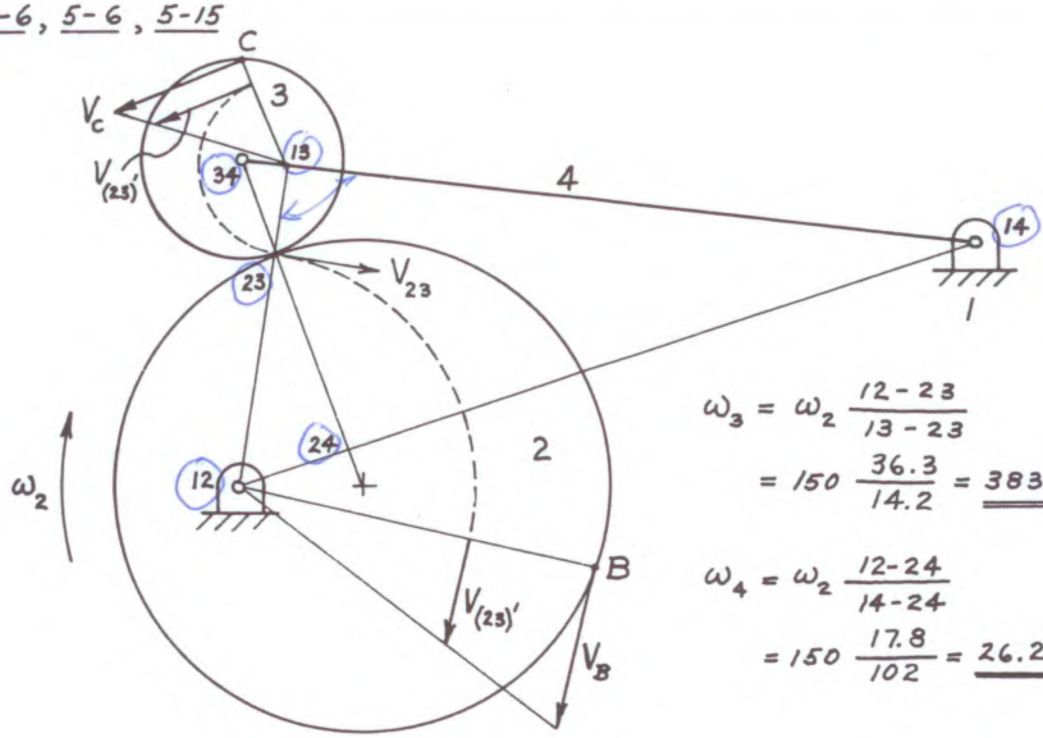
$OC = OB + BC$

$= 28.6 + 52.5$

$= 81.1$ mm

$$\frac{\omega_3}{\omega_4} = \frac{(81.1)\,2}{28.6} = \underline{5.67}$$

4-6, 5-6, 5-15



$\frac{4(4-1)}{2} = 6$

$$\omega_3 = \omega_2 \, \frac{12-23}{13-23}$$

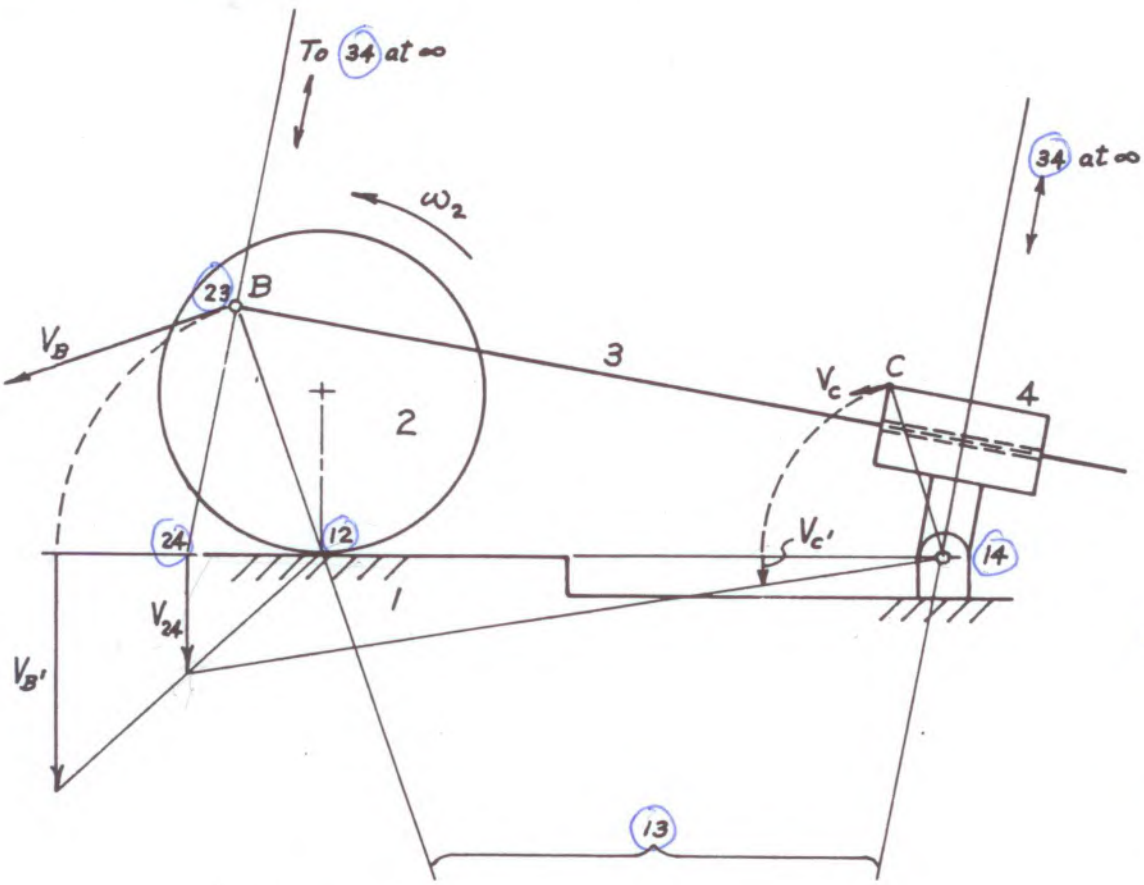
$$= 150 \, \frac{36.3}{14.2} = \underline{383 \text{ r/min ccw}}$$

$$\omega_4 = \omega_2 \, \frac{12-24}{14-24}$$

$$= 150 \, \frac{17.8}{102} = \underline{26.2 \text{ r/min ccw}}$$

CHAPTER 4. INSTANT CENTERS
CHAPTER 5. VELOCITIES BY INSTANT CENTERS AND BY COMPONENTS

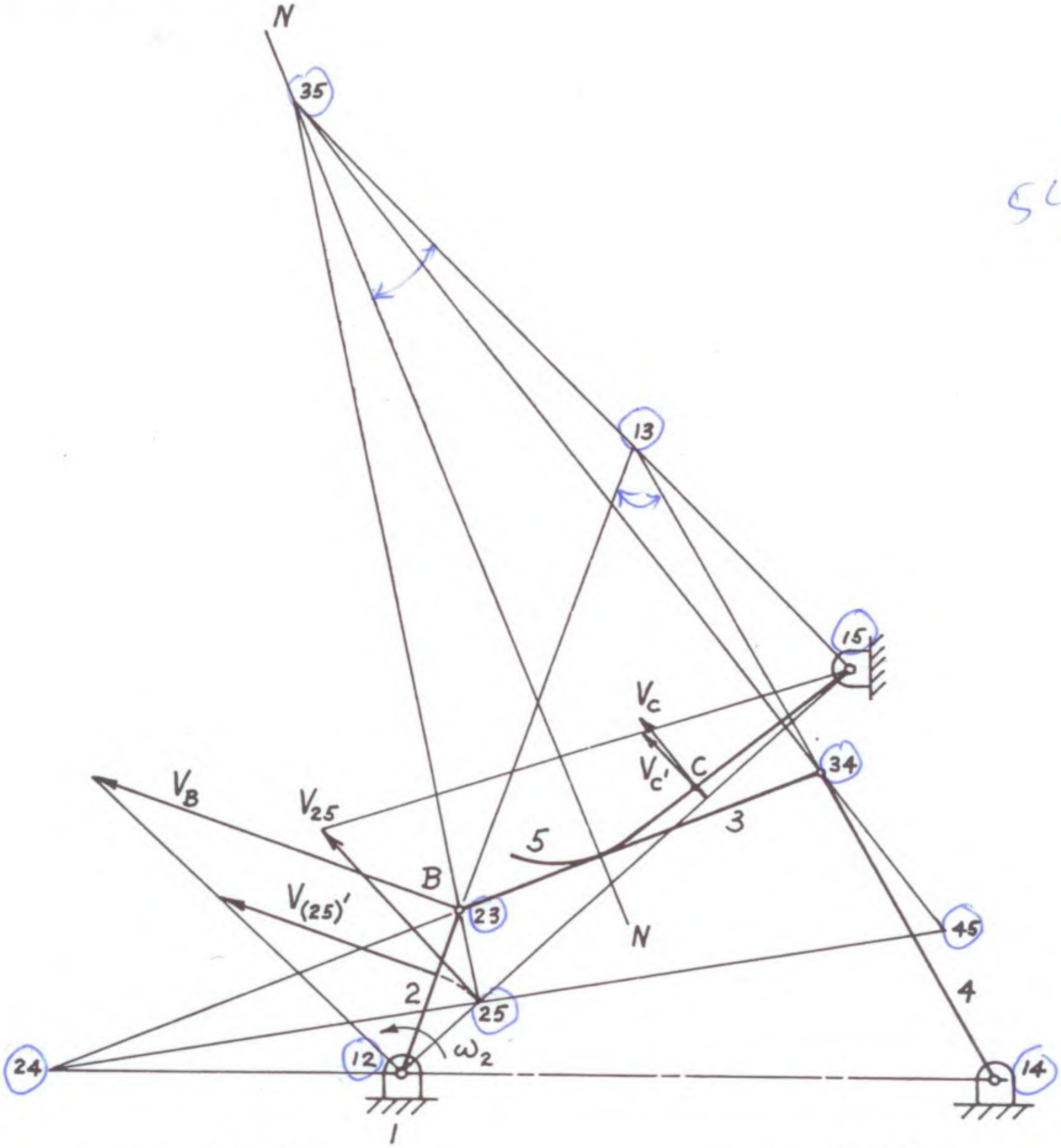
4-7, 5-7, 5-16



$\frac{4(4-1)}{2} = 6$

$$\omega_4 = \omega_2 \frac{12-24}{14-24} = 120 \frac{20.8}{117} = \underline{\underline{21.3 \text{ r/min ccw}}}$$

4-8, 5-8, 5-17



$5(5-17) = w$
$/2$

$$\omega_3 = \omega_2 \frac{12-23}{13-23}$$
$$= 75 \frac{28.6}{80.3} = \underline{26.7 \text{ r/min cw}}$$

$$\omega_4 = \omega_2 \frac{12-24}{14-24}$$
$$= 75 \frac{57.9}{153} = \underline{28.4 \text{ r/min ccw}}$$

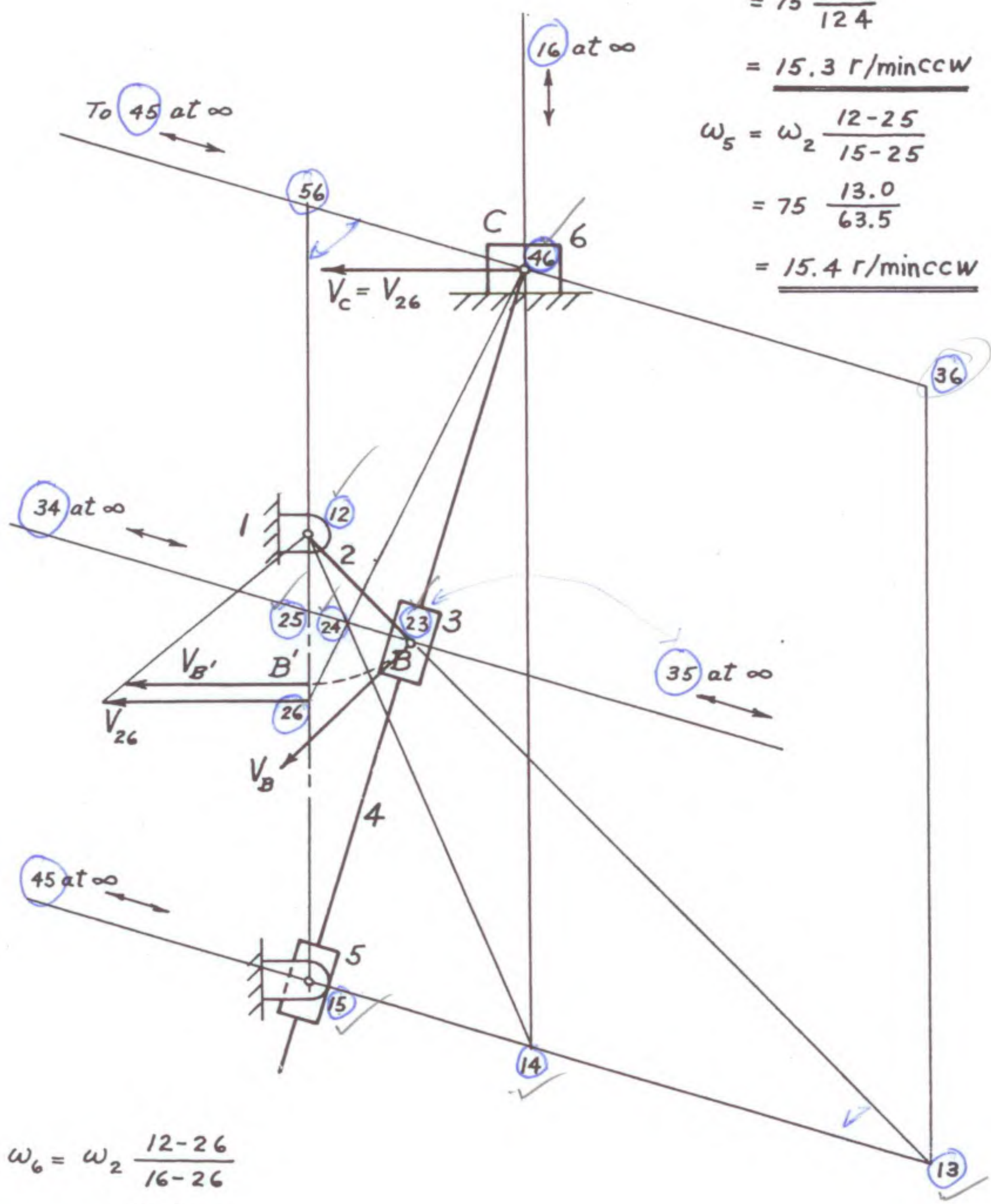
$$\omega_5 = \omega_2 \frac{12-25}{15-25}$$
$$= 75 \frac{17.3}{80.5}$$
$$= \underline{16.1 \text{ r/min cw}}$$

4-9, 5-9, 5-18

6 C 6-11 1/2 = 15

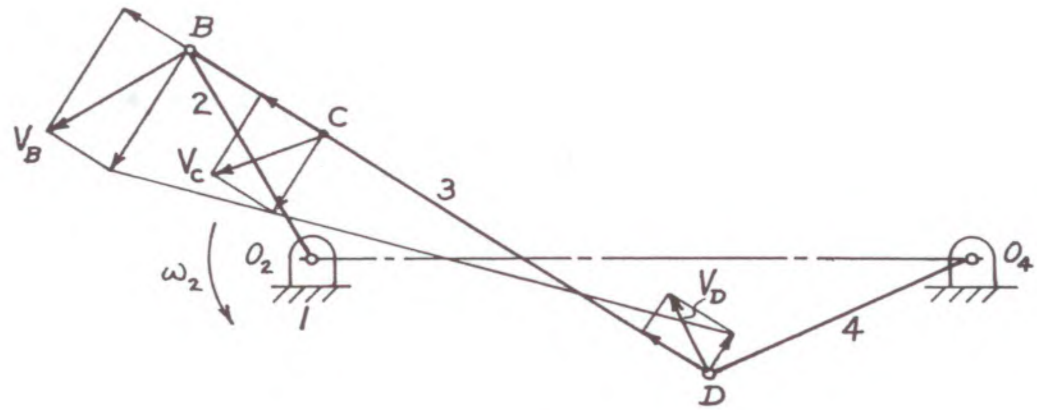
$$\omega_3 = \omega_2 \frac{12-23}{13-23} = 75 \frac{25.4}{124} = \underline{15.3 \text{ r/min ccw}}$$

$$\omega_5 = \omega_2 \frac{12-25}{15-25} = 75 \frac{13.0}{63.5} = \underline{15.4 \text{ r/min ccw}}$$

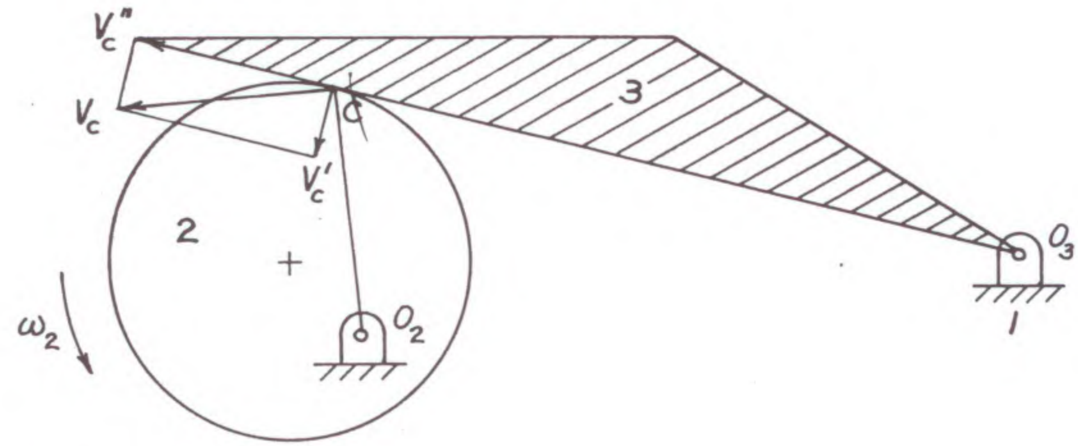


$$\omega_6 = \omega_2 \frac{12-26}{16-26} = 75 \frac{27.9}{\infty} = \underline{0}$$

5-19



5-20



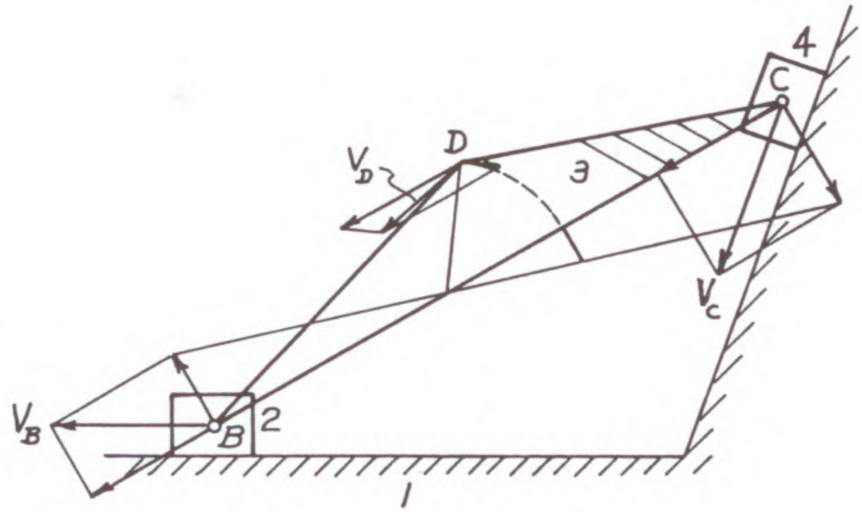
$$V_C = (O_2C)\,\omega_2$$

$$= \frac{1.54}{12}\;\frac{2\pi(100)}{60} = 1.34\ ft/s$$

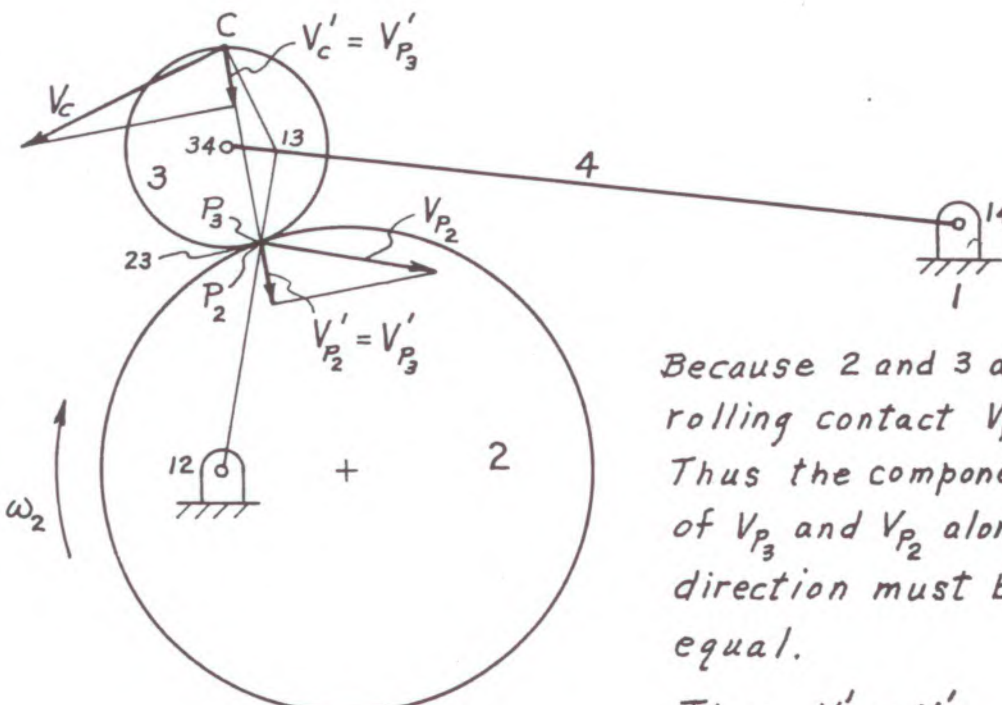
V_C' scales $0.46\ ft/s$; sliding vel. V_C'' scales $\underline{1.27\ ft/s}$

$$\omega_3 = \frac{V_C'}{O_3C} = \frac{0.46(60)}{\frac{4.28}{12}(2\pi)} = \underline{12.3\ r/min\ ccw}$$

5-21



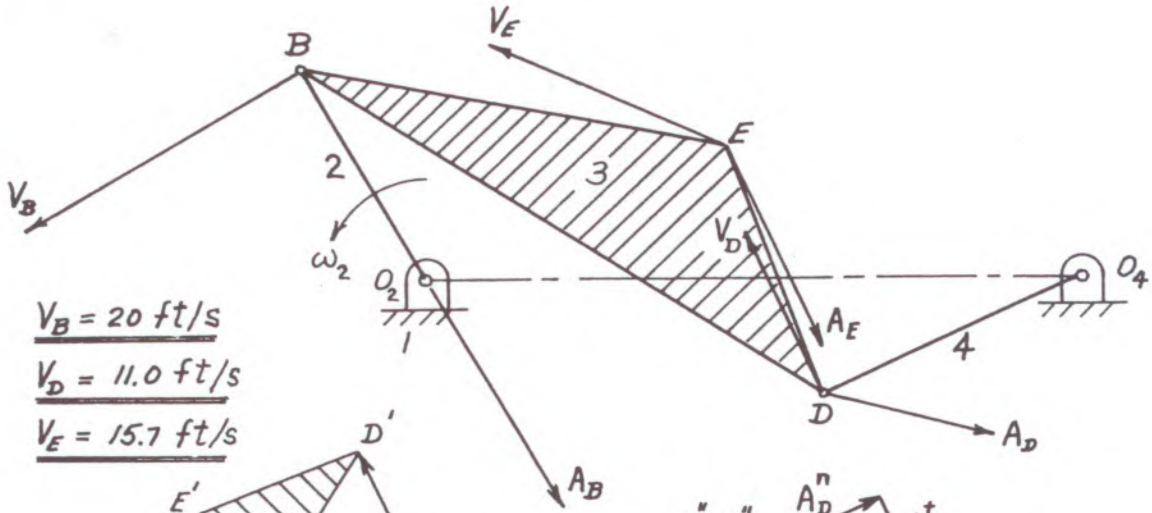
5-22



Because 2 and 3 are in rolling contact $V_{P_3} = V_{P_2}$. Thus the components of V_{P_3} and V_{P_2} along any direction must be equal.

Thus $V_{P_3}' = V_{P_2}'$

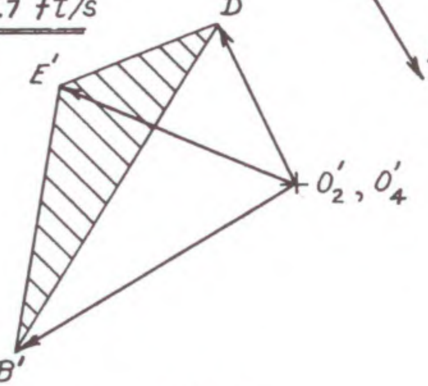
<u>6-1</u>, <u>7-1</u>



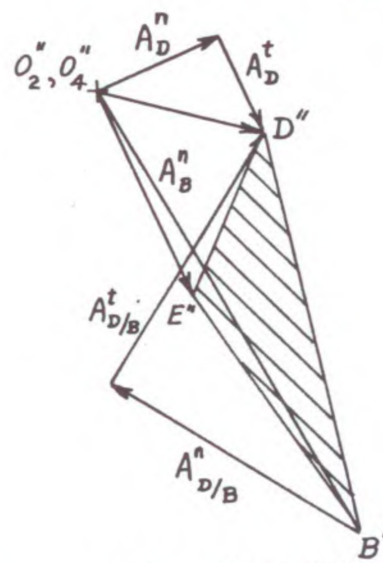
$\underline{V_B = 20 \text{ ft/s}}$

$\underline{V_D = 11.0 \text{ ft/s}}$

$\underline{V_E = 15.7 \text{ ft/s}}$



1 in. = 10 ft/s



1 in. = 1000 ft/s

(i) $\bar{V}_D = \bar{V}_{D/B} + \bar{V}_B$

(ii) $\bar{V}_E = \bar{V}_{E/B} + \bar{V}_B$

$\bar{V}_E = \bar{V}_{E/D} + \bar{V}_D$

$\bar{V}_{E/B} + \bar{V}_B = \bar{V}_{E/D} + \bar{V}_D$

$$\omega_2 = \frac{V_B}{O_2B} = \frac{20}{1.5/12} = \underline{160 \text{ rad/s ccw}}$$

$$\omega_3 = \frac{V_{D/B}}{BD} = \frac{23.9}{3.75/12} = \underline{76.6 \text{ rad/s ccw}}$$

$$\omega_4 = \frac{V_D}{O_4D} = \frac{11}{1.75/12} = \underline{75.3 \text{ rad/s cw}}$$

$$A_D^n +\!\!\!\rightarrow A_D^t = A_B^n +\!\!\!\rightarrow A_B^t +\!\!\!\rightarrow A_{D/B}^n +\!\!\!\rightarrow A_{D/B}^t$$

$$A_D^n = \frac{V_D^2}{O_4D} = \frac{(11.0)^2}{1.75/12} = 830 \text{ ft/s}^2$$

$$A_B^n = \frac{V_B^2}{O_2B} = \frac{(20)^2}{1.5/12} = 3200 \text{ ft/s}^2$$

$$A_{D/B}^n = \frac{V_{D/B}^2}{BD} = \frac{(23.9)^2}{3.75/12} = 1825 \text{ ft/s}^2$$

$\underline{A_B = 3200 \text{ ft/s}^2}$

$\underline{A_D = 1060 \text{ ft/s}^2}$

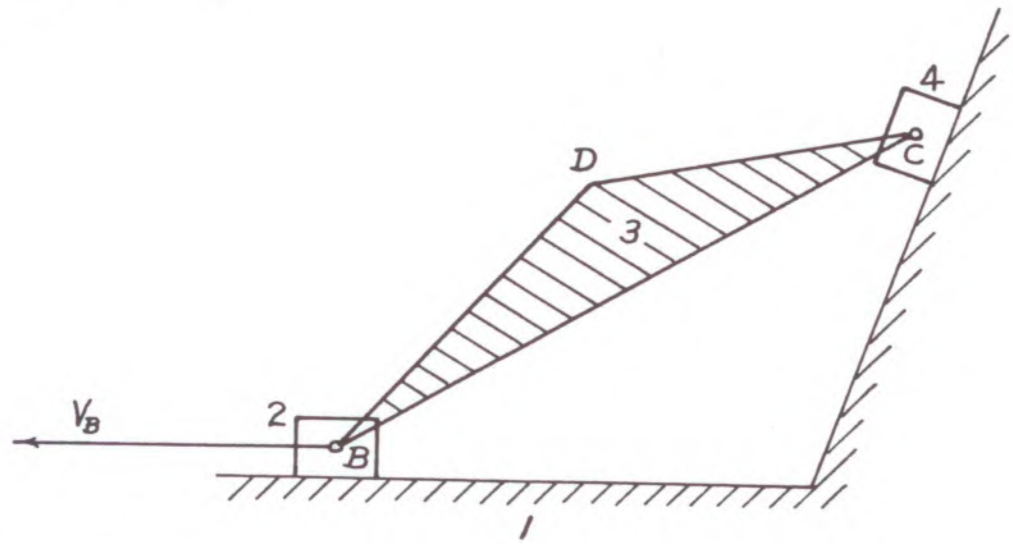
$\underline{A_E = 1390 \text{ ft/s}^2}$

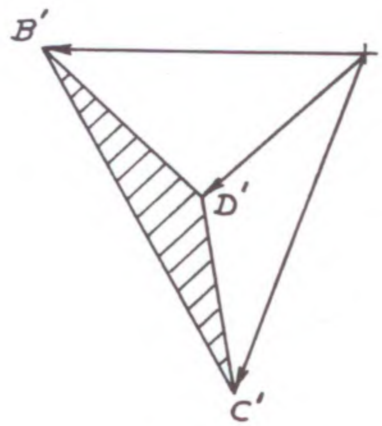
$$\alpha_3 = \frac{A_{D/B}^t}{BD} = \frac{1810}{3.75/12} = \underline{5800 \text{ rad/s ccw}}$$

$$\alpha_4 = \frac{A_D^t}{O_4D} = \frac{670}{1.75/12} = \underline{4590 \text{ rad/s ccw}}$$

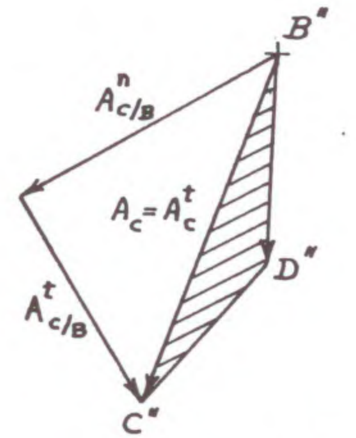
CHAPTER 6. VELOCITIES IN MECHANISMS BY METHOD OF RELATIVE VELOCITIES
CHAPTER 7. ACCELERATIONS IN MECHANISMS

<u>6-2</u>, <u>7-2</u>





$1\,mm = 0.120\ m/s$



$1\,mm = 12.0\ m/s^2$

$$\omega_3 = \frac{V_{C/B}}{BC} = \frac{7.44}{0.102} = \underline{72.9\ \mathbf{rad/s}\ \text{CW}}$$

$$\overset{\circ}{A^n_C} \rightarrow\!\!+ \overset{-\checkmark}{A^t_C} = \overset{\circ}{A^n_B} \rightarrow\!\!+ \overset{\circ}{A^t_B} \rightarrow\!\!+ \overset{\checkmark\checkmark}{A^n_{C/B}} \rightarrow\!\!+ \overset{-\checkmark}{A^t_{C/B}}$$

$$A^n_{C/B} = \frac{V^2_{C/B}}{BC} = \frac{(7.44)^2}{0.102} = 543\ m/s^2$$

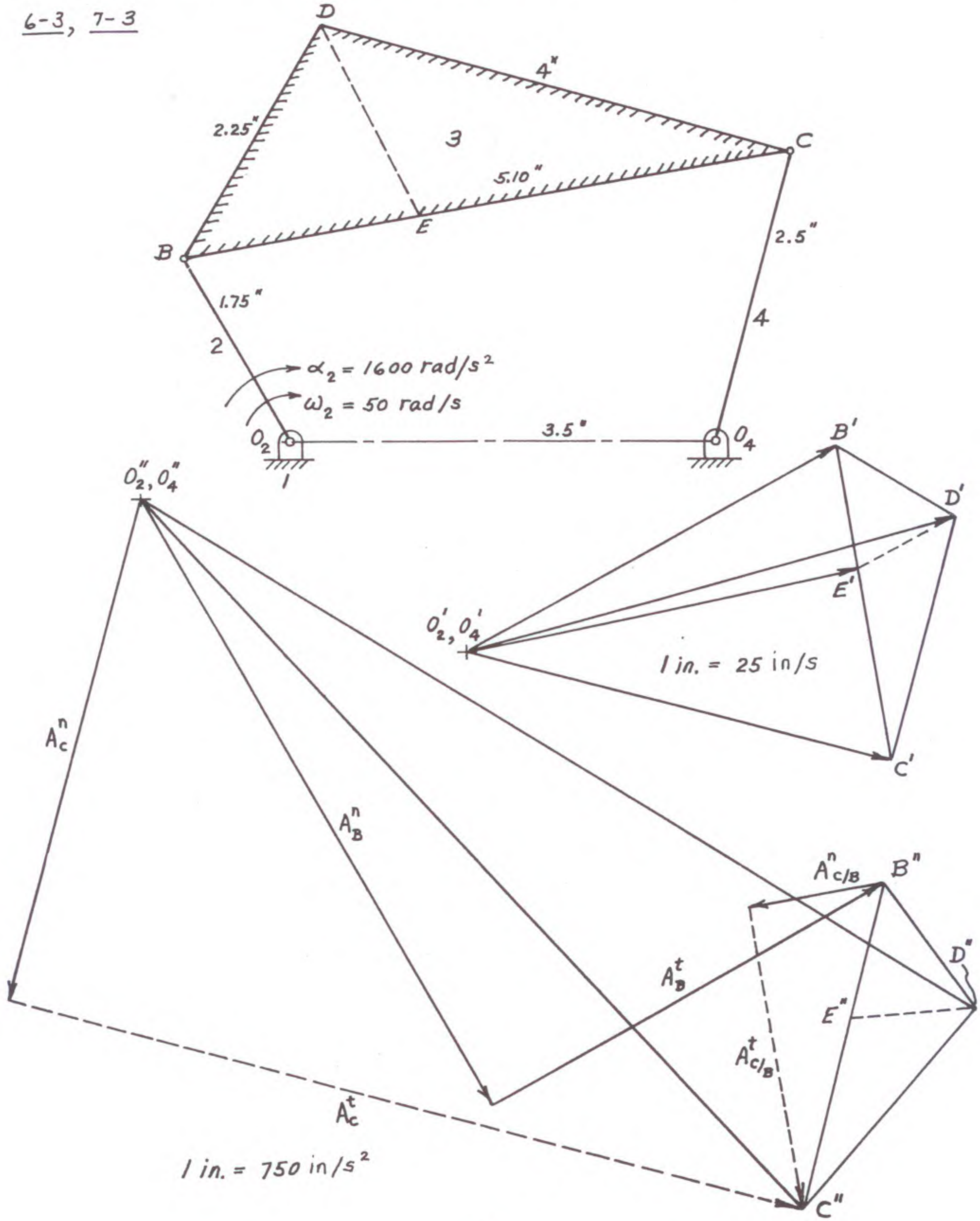
A_C scales $\underline{713\ m/s^2}$

A_D scales $\underline{402\ m/s^2}$

$$\alpha_3 = \frac{A^t_{C/B}}{BC} = \frac{457}{0.102}$$

$$= \underline{4\,480\ \mathbf{rad/s^2}\ \text{CW}}$$

6-3, 7-3


D
4"
2.25"
3
C
5.10"
E
2.5"
B
1.75"
4
2
$\alpha_2 = 1600$ rad/s²
$\omega_2 = 50$ rad/s
O_2
3.5"
O_4
1
B'
O_2'', O_4''
D'
E'
O_2', O_4'
1 in. = 25 in/s
C'
A_C^n
A_B^n
$A_{C/B}^n$
B''
D''
A_B^t
E''
$A_{C/B}^t$
A_C^t
1 in. = 750 in/s²
C''

6-3 (CONT.), 7-3 (CONT.)

$V_B = 1.75(50) = 87.5$ in/s $\qquad V_C$ scales 90 in/s

$V_{C/B}$ scales 66.5 in/s $\qquad \omega_3 = \dfrac{V_{C/B}}{BC} = \dfrac{66.5}{5.10} = \underline{13.02 \text{ rad/s cw}}$

$$\overset{\checkmark\checkmark}{A_C^n} \rightarrow\!+ \overset{-\checkmark}{A_C^t} = \overset{\checkmark\checkmark}{A_B^n} \rightarrow\!+ \overset{\checkmark\checkmark}{A_B^t} \rightarrow\!+ \overset{\checkmark\checkmark}{A_{C/B}^n} \rightarrow\!+ \overset{-\checkmark}{A_{C/B}^t}$$

$$A_B^n = \frac{V_B^2}{O_2B} = \frac{(87.5)^2}{1.75} = \frac{7650}{1.75} = 4365 \text{ in/s}^2$$

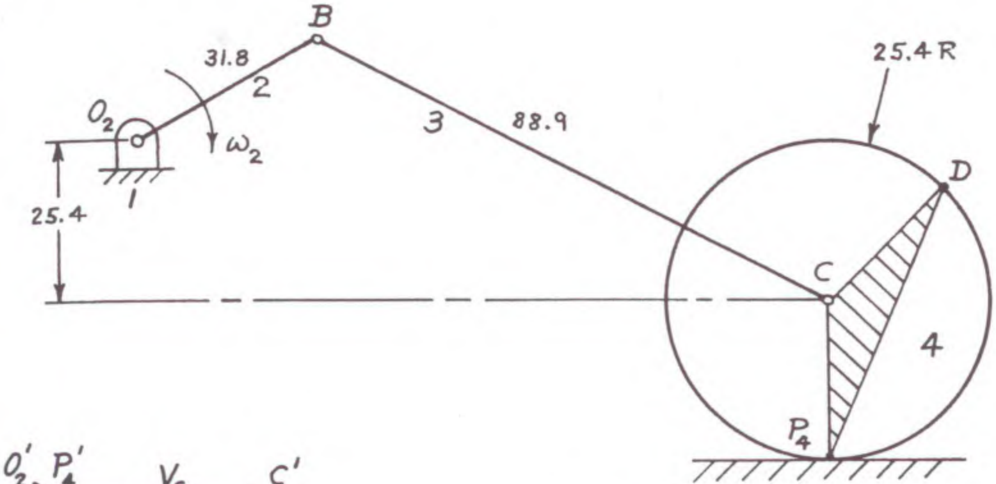
$$A_B^t = (O_2B)\alpha_2 = 1.75(1600) = 2800 \text{ in/s}^2$$

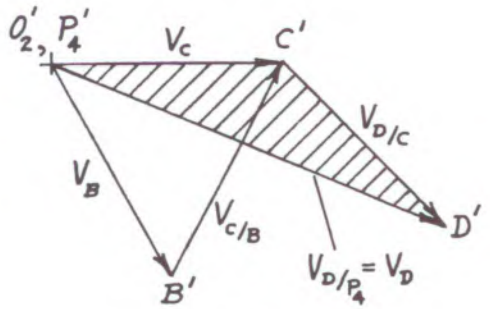
$$A_C^n = \frac{V_C^2}{O_4C} = \frac{(90)^2}{2.5} = 3240 \text{ in/s}^2, \qquad A_{C/B}^n = \frac{V_{C/B}^2}{BC} = \frac{(66.5)^2}{5.10} = 865 \text{ in/s}^2$$

$A_{C/B}^t$ scales 1920 in/s², $\qquad A_C^t$ scales 5090 in/s²

$$\alpha_3 = \frac{A_{C/B}^t}{BC} = \frac{1920}{5.10} = \underline{376 \text{ rad/s}^2 \text{ cw}} \qquad \alpha_4 = \frac{A_C^t}{O_4C} = \frac{5090}{2.5} = \underline{2040 \text{ rad/s}^2 \text{ cw}}$$

6-4, 7-9





$$V_B = (O_2B)\,\omega_2 = 0.0318(144) = 4.58\ \text{m/s}$$

$$\overset{-\checkmark}{\bar{V}_C} = \overset{\checkmark\checkmark}{\bar{V}_B} \rightarrow \overset{-\checkmark}{\bar{V}_{C/B}}$$

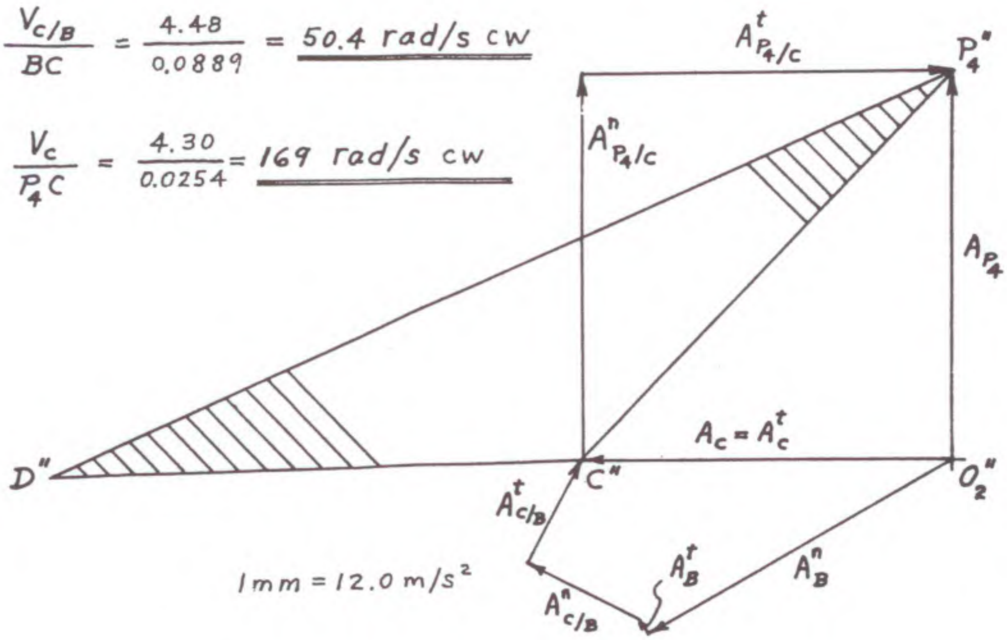
$$\overset{--}{\bar{V}_D} = \overset{\checkmark\checkmark}{\bar{V}_C} \rightarrow \overset{-\checkmark}{\bar{V}_{D/C}}$$

$$\overset{--}{\bar{V}_D} = \overset{\circ}{\bar{V}_{P_4}} \rightarrow \overset{-\checkmark}{\bar{V}_{D/P_4}}$$

1 mm = 0.120 m/s

$$\omega_3 = \frac{V_{C/B}}{BC} = \frac{4.48}{0.0889} = \underline{\underline{50.4\ \text{rad/s cw}}}$$

$$\omega_4 = \frac{V_C}{P_4C} = \frac{4.30}{0.0254} = \underline{\underline{169\ \text{rad/s cw}}}$$



1 mm = 12.0 m/s²

7-9 (CONT.)

$$A_B^n = (O_2B)\omega_2^2 = 0.0318\,(144)^2 = 659\text{ m/s}^2$$

$$A_B^t = (O_2B)\alpha_2 = 0.0318\,(1000) = 31.8\text{ m/s}^2$$

$$A_C^n \overset{\rightarrow}{+} A_C^t = A_B^n \overset{\rightarrow}{+} A_B^t \overset{\rightarrow}{+} A_{C/B}^n \overset{\rightarrow}{+} A_{C/B}^t$$

$$A_{C/B}^n = \frac{V_{C/B}^2}{BC} = \frac{(4.48)^2}{0.0889} = 226\text{ m/s}^2$$

$$A_{P_4/C}^n = \frac{V_{P_4/C}^2}{P_4C} = \frac{(4.30)^2}{0.0254} = 728\text{ m/s}^2$$

$$A_{P_4} = A_{P_4/C}^n$$

D″ located by image method.

$$\alpha_3 = \frac{A_{C/B}^t}{BC} = \frac{213}{0.0889}$$

$$= \underline{\underline{2396\text{ rad/s}^2\text{ ccw}}}$$

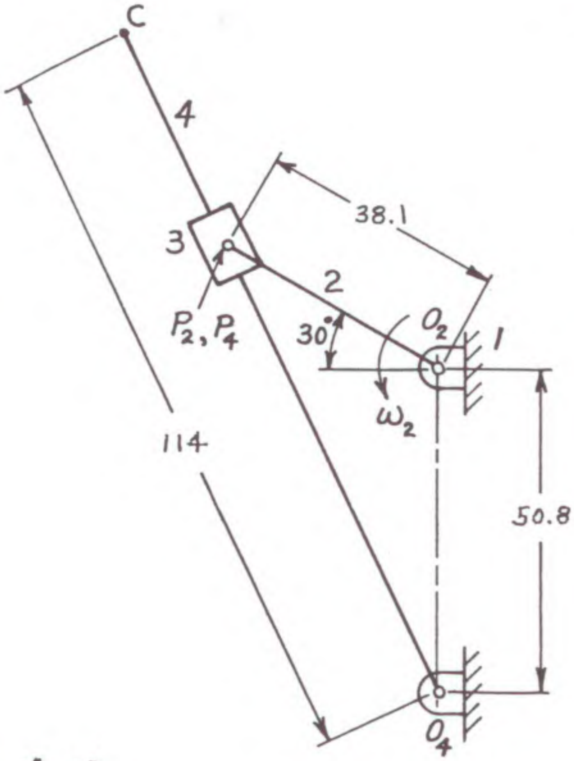
$$\alpha_4 = \frac{A_{P_4/C}^t}{P_4C} = \frac{692}{0.0254}$$

$$= \underline{\underline{27{,}200\text{ rad/s}^2\text{ ccw}}}$$

CHAPTER 6. VELOCITIES IN MECHANISMS BY METHOD OF RELATIVE VELOCITIES

CHAPTER 7. ACCELERATIONS IN MECHANISMS

6-5, 7-10



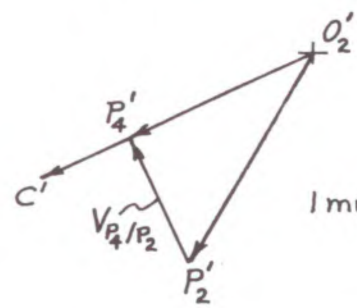
$$V_{P_2} = (O_2P_2)\,\omega_2 = 0.0381(120) = 4.57 \text{ m/s}$$

$$\overset{-\checkmark}{V_{P_4}} = \overset{\checkmark\checkmark}{V_{P_2}} \;+\!\!\!\rightarrow\; \overset{-\checkmark}{V_{P_4/P_2}}$$

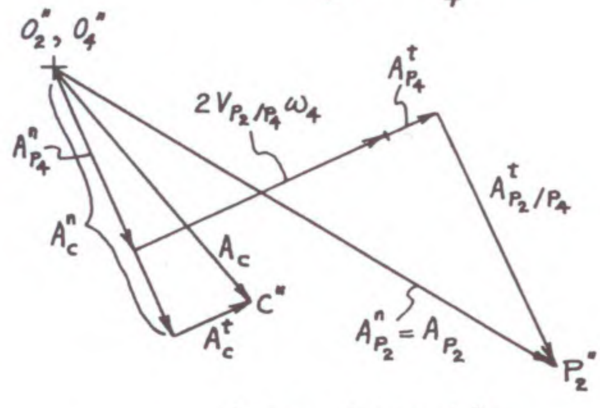
$$\frac{V_C}{V_{P_4}} = \frac{O_4C}{O_4P_4}, \quad V_C = \frac{O_4C}{O_4P_4} V_{P_4}$$

$$= \frac{114}{77.0} V_{P_4} = 1.48\, V_{P_4}$$

$$\omega_4 = \frac{V_{P_4}}{O_4P_4} = \frac{3.81}{0.077} = \underline{49.5 \text{ rad/s ccw}}$$



1 mm = 0.120 m/s



1 mm = 6.00 m/s²

$$2V_{P_2/P_4}\,\omega_4 = 2(2.59)49.5 = 256 \text{ m/s}^2$$

$$A_C^n = \frac{V_C^2}{O_4C} = \frac{(5.64)^2}{0.114} = 279 \text{ m/s}^2$$

$$A_C^t = A_{P_4}^t \frac{O_4C}{O_4P_4} = 54.9\frac{0.114}{0.077} = 81.3 \text{ m/s}^2$$

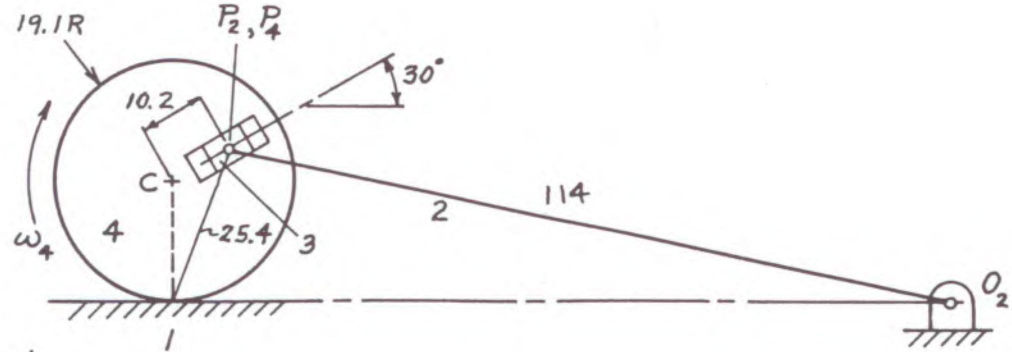
$$\alpha_4 = \frac{A_C^t}{O_4C} = \frac{81.3}{0.114} = \underline{713 \text{ rad/s}^2 \text{ cw}}$$

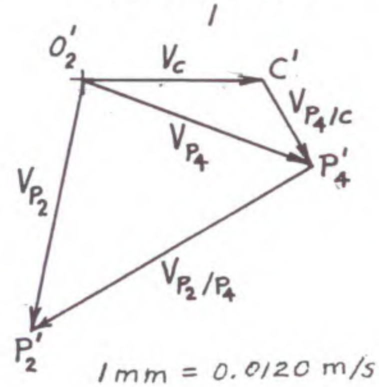
$$\overset{\checkmark\checkmark}{A_{P_2}^n} + \overset{\circ}{A_{P_2}^t} = \overset{\checkmark\checkmark}{A_{P_4}^n} + \overset{-\checkmark}{A_{P_4}^t} + \overset{\circ}{A_{P_2/P_4}^n} + \overset{-\checkmark}{A_{P_2/P_4}^t} + \overset{\checkmark\checkmark}{2V_{P_2/P_4}\omega_4}$$

$$A_{P_2}^n = (O_2P_2)\,\omega_2^2 = 0.0381(120)^2 = 549 \text{ m/s}^2$$

$$A_{P_4}^n = \frac{V_{P_4}^2}{O_4P_4} = \frac{(3.81)^2}{0.077} = 189 \text{ m/s}^2$$

6-6, 7-13





1 mm = 0.0120 m/s

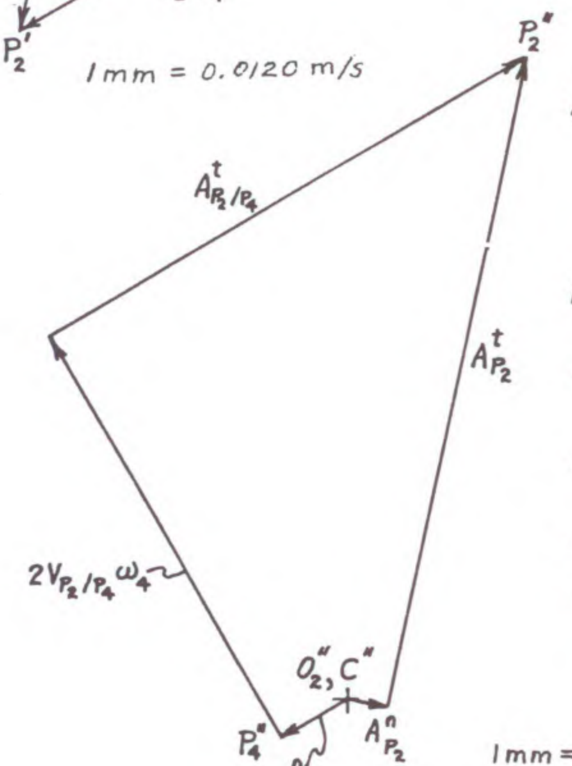
$$V_c = R\omega_4 = 0.0191(18) = 0.344 \text{ m/s}$$

$$V_{P_4} = 0.0254(18) = 0.457 \text{ m/s}$$

$$V_{P_2} = V_{P_4} \rightarrow\!\!+ V_{P_2/P_4}$$

$$\omega_2 = \frac{V_{P_2}}{O_2P_2} = \frac{0.482}{0.114} = \underline{\underline{4.23 \text{ rad/s ccw}}}$$

$$\alpha_4 = 0$$



$$A^n_{P_2} \rightarrow\!\!+ A^t_{P_2} = A^n_C \rightarrow\!\!+ A^t_C \rightarrow\!\!+ A^n_{P_4/C} \rightarrow\!\!+ A^t_{P_4/C} \rightarrow\!\!+ A^n_{P_2/P_4} \rightarrow\!\!+ A^t_{P_2/P_4} \rightarrow\!\!+ 2V_{P_2/P_4}\omega_4$$

$$A^n_{P_2} = \frac{V^2_{P_2}}{O_2P_2} = \frac{(0.482)^2}{0.114} = 2.04 \text{ m/s}^2$$

$$A^n_{P_4/C} = \frac{V^2_{P_4/C}}{P_4C} = \frac{(0.189)^2}{0.0102} = 3.50 \text{ m/s}^2$$

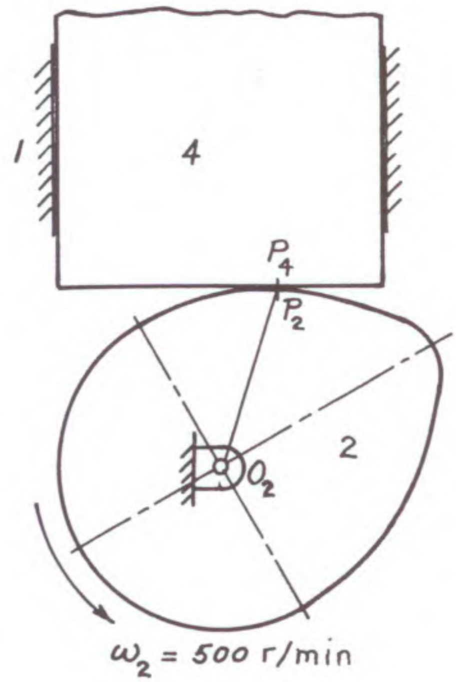
$$2V_{P_2/P_4}\omega_4 = 2\,(0.610)18 = 22.0 \text{ m/s}^2$$

$A^t_{P_2}$ scales 31.4 m/s^2

$$\alpha_2 = \frac{A^t_{P_2}}{O_2P_2} = \frac{31.4}{0.114} = \underline{275 \text{ rad/s}^2 \text{ cw}}$$

1 mm = 0.300 m/s²

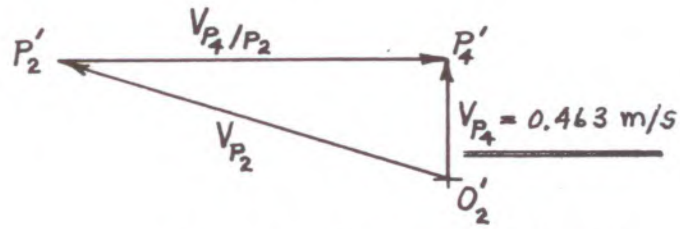
6-7



$$\omega_2 = 500\,\frac{2\pi}{60} = 52.4 \text{ rad/s}$$

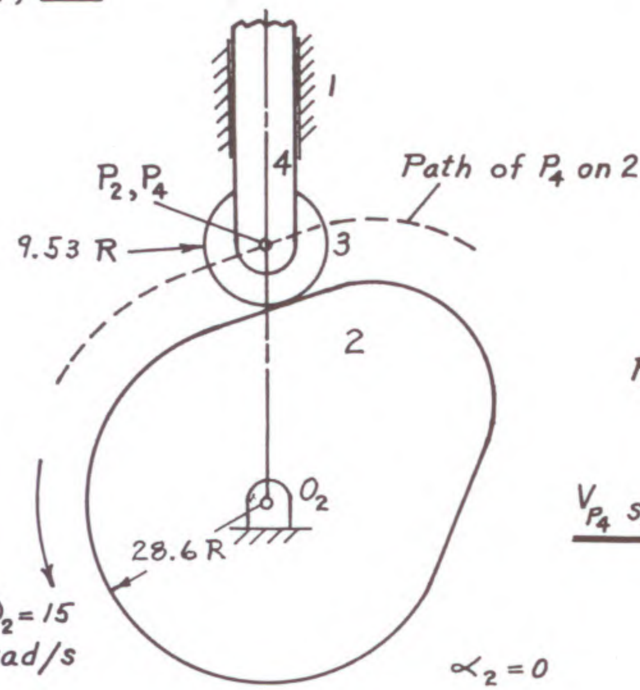
$$V_{P_2} = (O_2P_2)\,\omega_2 = 0.0292(52.4) = 1.53 \text{ m/s}$$

$$\overset{-\checkmark}{\vec{V}_{P_4}} = \overset{\checkmark\checkmark}{\vec{V}_{P_2}} \;\text{+}\!\!\!\rightarrow\; \overset{-\checkmark}{\vec{V}_{P_4/P_2}}$$



Scale: 1 mm = 0.0240 m/s

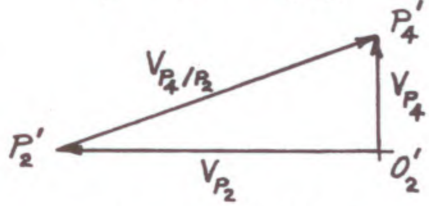
<u>6-8</u>, <u>7-11</u>



$\alpha_2 = 0$

$$V_{P_2} = (O_2P_2)\omega_2$$
$$= 0.0406(15) = 0.610 \text{ m/s}$$

$$\overset{-\checkmark}{V_{P_4}} = \overset{\checkmark\checkmark}{V_{P_2}} \mathbin{+\!\!\!\!\rightarrow} \overset{-\checkmark}{V_{P_4/P_2}}$$



1 mm = 0.012 m/s

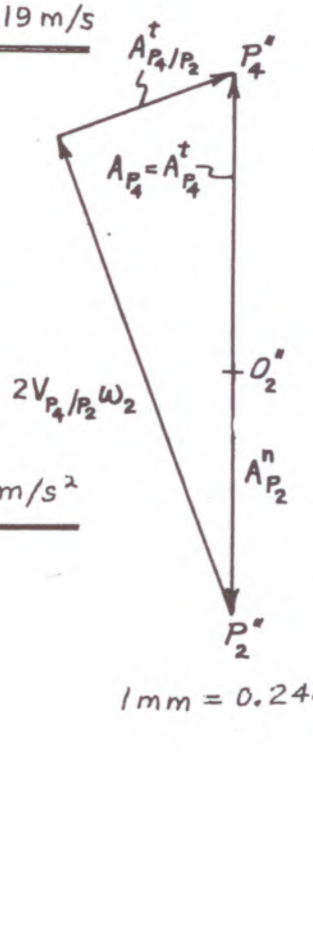
<u>V_{P_4} scales 0.219 m/s</u>

$$\overset{\circ}{A^n_{P_4}} \mathbin{+\!\!\!\!\rightarrow} \overset{-\checkmark}{A^t_{P_4}} = \overset{\checkmark\checkmark}{A^n_{P_2}} \mathbin{+\!\!\!\!\rightarrow} \overset{\circ}{A^t_{P_2}} \mathbin{+\!\!\!\!\rightarrow} \overset{\circ}{A^n_{P_4/P_2}} \mathbin{+\!\!\!\!\rightarrow} \overset{-\checkmark}{A^t_{P_4/P_2}} \mathbin{+\!\!\!\!\rightarrow} \overset{\checkmark\checkmark}{2V_{P_4/P_2}\omega_2}$$

$$A^n_{P_2} = (O_2P_2)\,\omega_2^2 = 0.0406(15)^2 = 9.14 \text{ m/s}^2$$

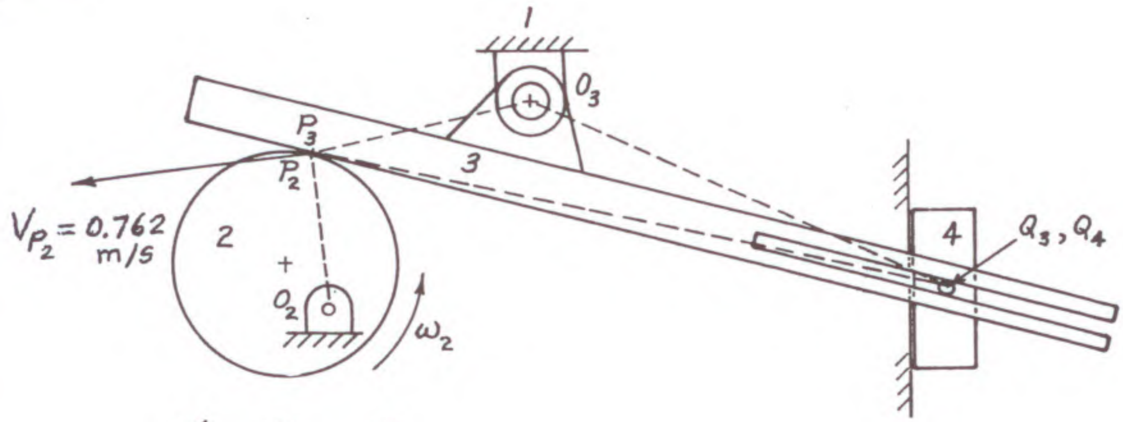
$$2V_{P_4/P_2}\,\omega_2 = 2(0.640)15 = 19.2 \text{ m/s}^2$$

<u>A_{P_4} scales 11.2 m/s^2</u>



1 mm = 0.240 m/s^2

6-9



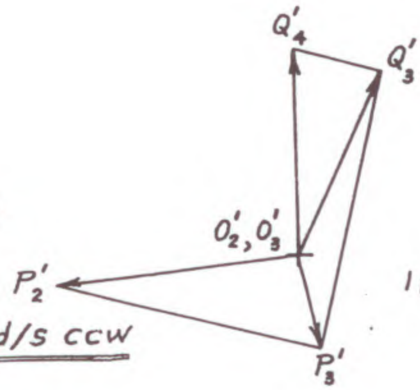
$$\vec{V}_{P_3} = \vec{V}_{P_2} +\!\!\!\rightarrow \vec{V}_{P_3/P_2}$$

$$\vec{V}_{Q_3} = \vec{V}_{P_3} +\!\!\!\rightarrow \vec{V}_{Q_3/P_3}$$

$$\vec{V}_{Q_4} = \vec{V}_{Q_3} +\!\!\!\rightarrow \vec{V}_{Q_4/Q_3}$$

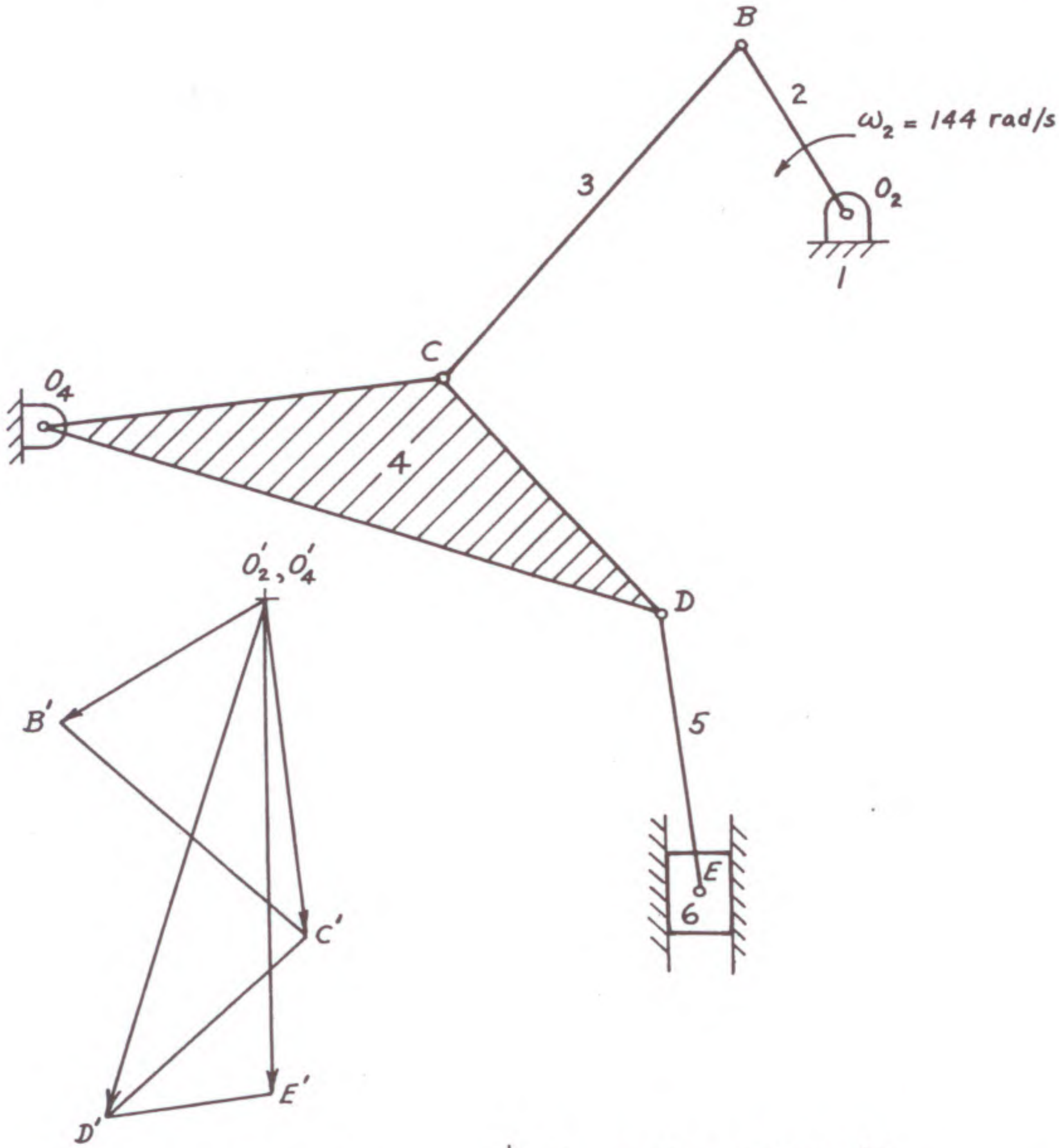
V_{Q_4} scales $\underline{\underline{0.640 \text{ m/s}}}$

$$\omega_3 = \frac{V_{P_3}}{O_3P_3} = \frac{0.305}{0.0349} = \underline{\underline{8.74 \text{ rad/s ccw}}}$$



1mm = 0.020 m/s

6-10



1 mm = 0.120 m/s

$V_B = (O_2B)\omega_2 = 0.0318(144) = 4.579$ m/s

$\overset{-\checkmark}{V_C} = \overset{\checkmark\checkmark}{V_B} \mapsto \overset{-\checkmark}{V_{C/B}}$

$\overset{-\checkmark}{V_D} = \overset{\checkmark\checkmark}{V_C} \mapsto \overset{-\checkmark}{V_{D/C}}$

$\overset{-\checkmark}{V_E} = \overset{\checkmark\checkmark}{V_D} \mapsto \overset{-\checkmark}{V_{E/D}}$

V_E scales 9.45 m/s

$\omega_3 = \dfrac{V_{C/B}}{BC} = \dfrac{6.22}{0.0699} = \underline{\underline{89.0 \text{ rad/s ccw}}}$

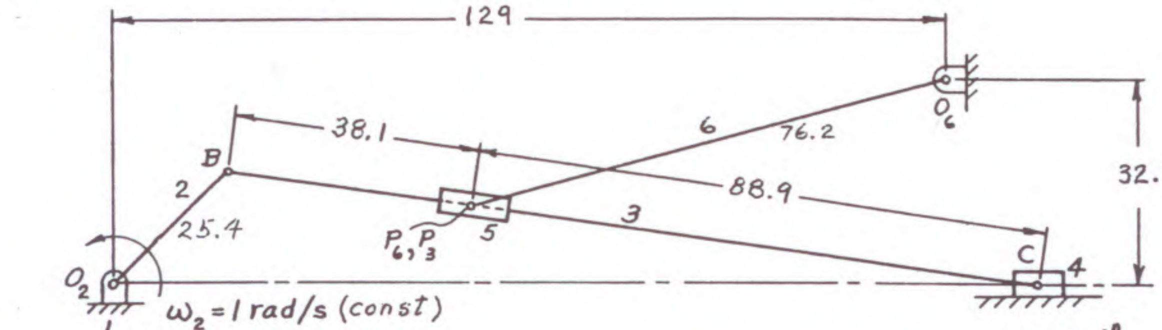
$\omega_4 = \dfrac{V_{D/O_4}}{O_4D} = \dfrac{10.4}{0.102} = \underline{\underline{102 \text{ rad/s cw}}}$

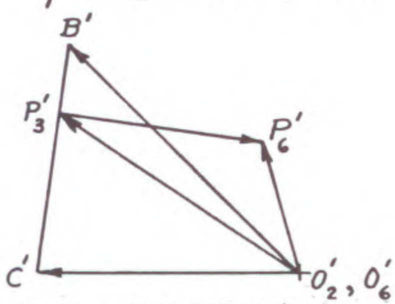
$\omega_5 = \dfrac{V_{D/E}}{DE} = \dfrac{3.29}{0.0445} = \underline{\underline{73.9 \text{ rad/s ccw}}}$

CHAPTER 6. VELOCITIES IN MECHANISMS BY METHOD OF RELATIVE VELOCITIES

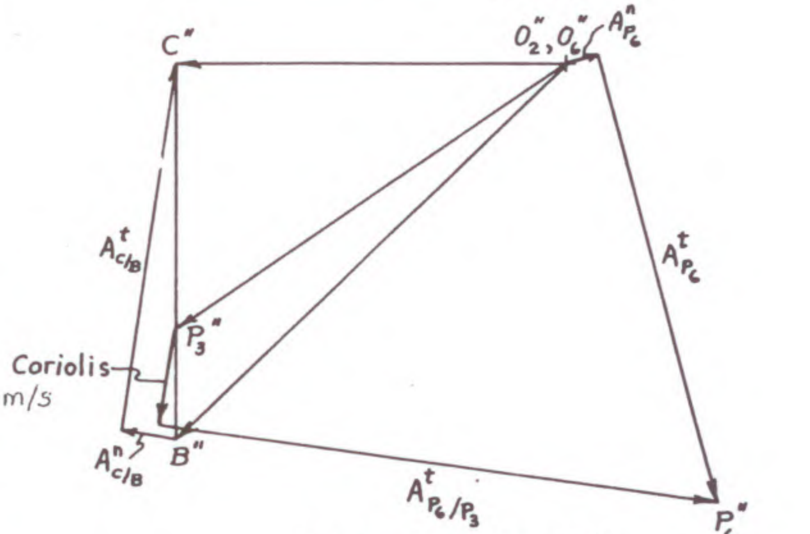
CHAPTER 7. ACCELERATIONS IN MECHANISMS

6-11, 7-14





1 mm = 0.0005 m/s



1 mm = 0.0003 m/s²

$V_B = (O_2B)\,\omega_2 = 0.0254(1) = 0.0254$ m/s

$V_{B/C}$ scales 0.0180 m/s

P_3' located by image method.

V_{P_6} scales 0.0104 m/s

V_{P_6/P_3} scales 0.0163 m/s

$$\omega_3 = \frac{V_{B/C}}{BC} = \frac{0.0180}{0.127} = \underline{0.142 \text{ rad/s cw}}$$

$$\omega_6 = \frac{V_{P_6}}{O_6P_6} = \frac{0.0104}{0.0762} = \underline{0.136 \text{ rad/s cw}}$$

$$A^n_C + A^t_C = A^n_B + A^t_B + A^n_{C/B} + A^t_{C/B} \qquad (1)$$

$$A^n_B = \frac{V_B^2}{O_2B} = \frac{(0.0254)^2}{0.0254} = 0.0254 \text{ m/s}^2$$

$$A^n_{C/B} = \frac{V_{C/B}^2}{BC} = \frac{(0.0180)^2}{0.127} = 0.00255 \text{ m/s}^2$$

The vector diagram for Eq. (1) is then drawn. Next P_3'' is located by image method i.e.

$$B''P_3'' = \frac{BP_3}{BC}(B''C'')$$

$$A^n_{P_6} + A^t_{P_6} = A_{P_3} + A^n_{P_6/P_3} + A^t_{P_6/P_3} + 2V_{P_6/P_3}\omega_3 \qquad (2)$$

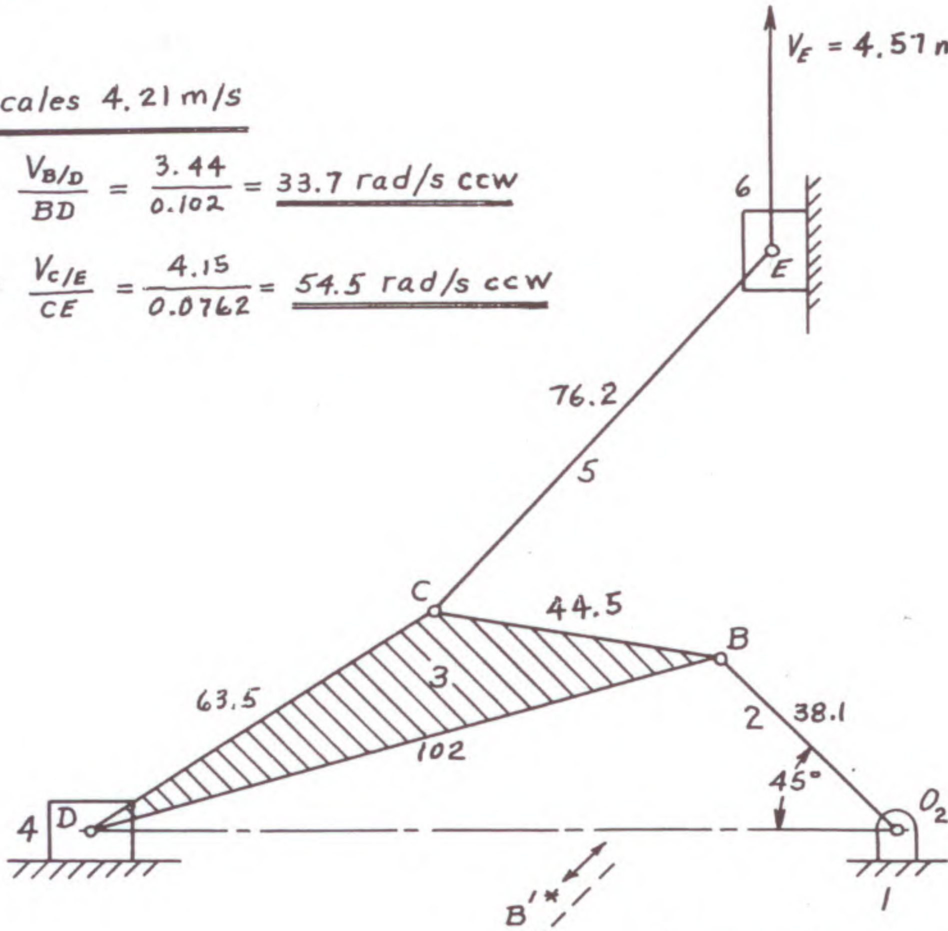
$$A^n_{P_6} = \frac{V_{P_6}^2}{O_6P_6} = \frac{(0.0104)^2}{0.0762} = 0.00142 \text{ m/s}^2$$

$$2V_{P_6/P_3}\omega_3 = 2(0.0163)0.142 = 0.00463 \text{ m/s}^2$$

$$\alpha_3 = \frac{A^t_{C/B}}{CB} = \frac{0.0175}{0.127} = \underline{0.138 \text{ rad/s}^2 \text{ ccw}}$$

$$\alpha_6 = \frac{A^t_{P_6/O_6}}{O_6P_6} = \frac{0.0218}{0.0762} = \underline{0.286 \text{ rad/s}^2 \text{ ccw}}$$

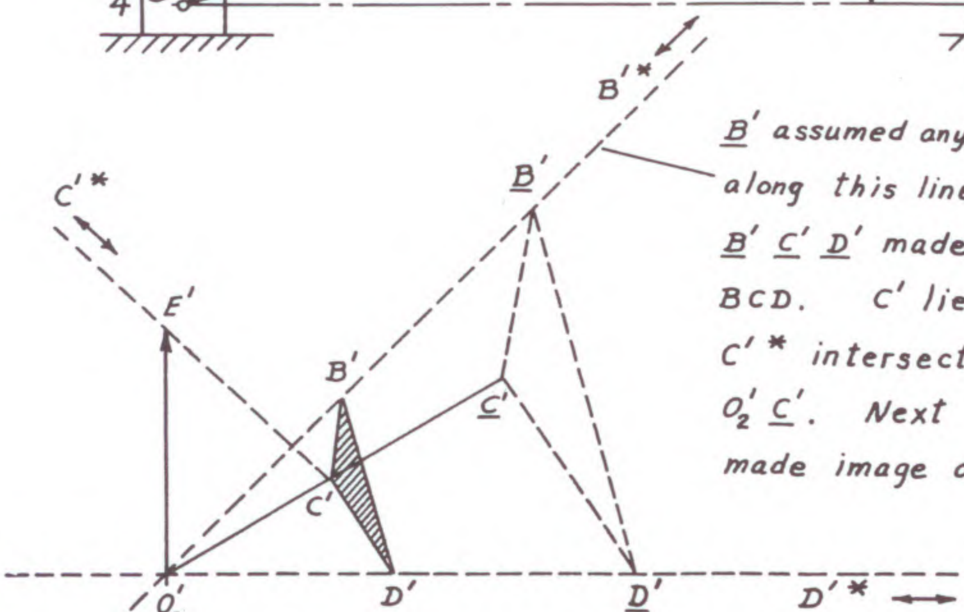
6-12, 7-5



V_D scales 4.21 m/s

$$\omega_3 = \frac{V_{B/D}}{BD} = \frac{3.44}{0.102} = \underline{33.7 \text{ rad/s ccw}}$$

$$\omega_5 = \frac{V_{C/E}}{CE} = \frac{4.15}{0.0762} = \underline{54.5 \text{ rad/s ccw}}$$



B' assumed anywhere along this line. Then B' C' D' made image of BCD. C' lies where C'* intersects line O_2' C'. Next B'C'D' made image of BCD.

1 mm = 0.120 m/s

7-5 (CONT.)

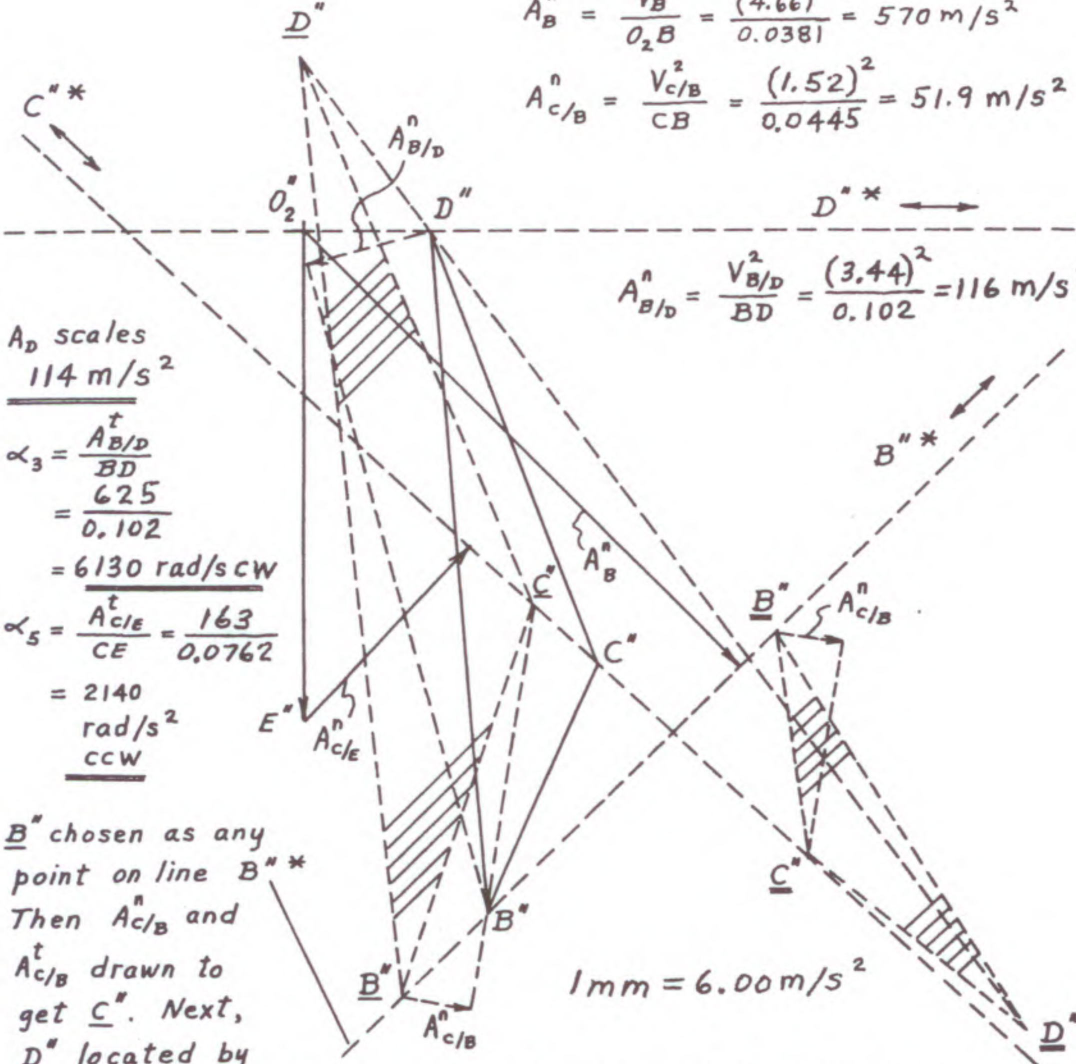
$$\bar{A}_C = A_E^n \nrightarrow A_E^t \nrightarrow A_{C/E}^n \nrightarrow A_{C/E}^t$$

$$A_{C/E}^n = \frac{V_{C/E}^2}{CE} = \frac{(4.15)^2}{0.0762} = 226 \text{ m/s}^2$$

$$A_B^n = \frac{V_B^2}{O_2B} = \frac{(4.66)^2}{0.0381} = 570 \text{ m/s}^2$$

$$A_{C/B}^n = \frac{V_{C/B}^2}{CB} = \frac{(1.52)^2}{0.0445} = 51.9 \text{ m/s}^2$$

$$A_{B/D}^n = \frac{V_{B/D}^2}{BD} = \frac{(3.44)^2}{0.102} = 116 \text{ m/s}^2$$



A_D scales
114 m/s^2

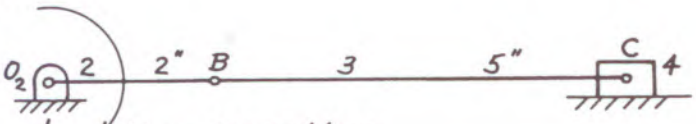
$$\alpha_3 = \frac{A_{B/D}^t}{BD} = \frac{625}{0.102} = 6130 \text{ rad/s CW}$$

$$\alpha_5 = \frac{A_{C/E}^t}{CE} = \frac{163}{0.0762} = 2140 \text{ rad/s}^2 \text{ CCW}$$

B'' chosen as any point on line B''* Then $A_{C/B}^n$ and $A_{C/B}^t$ drawn to get C''. Next, D'' located by making B'' C'' D'' the image of BCD. Next, B'' chosen anywhere on line B''*. Then $A_{C/B}^n$ and $A_{C/B}^t$ drawn to get C''. Next, D'' located by making B'' C'' D'' image of BCD. Next, where a line from D'' to D'' intersects D''* locates D''. Next, we locate B'' by laying off $A_{B/D}^n \nrightarrow A_{B/D}^t$ to locate B'' on line B''* Finally, C'' located by making D'' B'' C'' image of DBC.

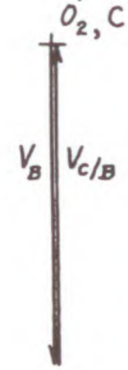
7-4

a)



$\omega_2 = 100$ rad/sec

$\alpha_2 = 0$



From velocity polygon $\underline{V_c = 0}$

$1\,mm = 0.100$ m/s

$V_B = (O_2B)\omega_2 = (0.0508)100 = 5.08$ m/s

$$\overset{-\checkmark}{V_c} = \overset{\checkmark\checkmark}{V_B} \;+\!\!\!\rightarrow\; \overset{-\checkmark}{V_{C/B}}$$

$$\overset{o}{A_C^n} \;+\!\!\!\rightarrow\; \overset{-\checkmark}{A_C^t} = \overset{\checkmark\checkmark}{A_B^n} \;+\!\!\!\rightarrow\; \overset{o}{A_B^t} \;+\!\!\!\rightarrow\; \overset{\checkmark\checkmark}{A_{C/B}^n} \;+\!\!\!\rightarrow\; \overset{-\checkmark}{A_{C/B}^t}$$

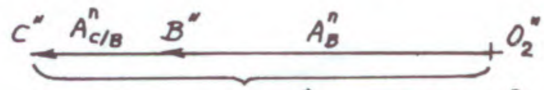
$$A_B^n = \frac{V_B^2}{O_2B} = \frac{(5.08)^2}{0.0508} = 508\ m/s^2$$

$$A_B^t = (O_2B)\alpha_2 = 0 \qquad A_{C/B}^n = \frac{V_{B/C}^2}{BC} = \frac{(5.08)^2}{0.127} = 203\ m/s^2$$

$$A^n = \frac{V^2}{R} = R\omega^2$$

$$A^t = R\alpha$$

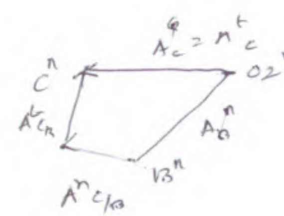
$$A = \sqrt{(A^n)^2 + (A^t)^2}$$



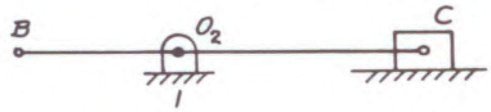
$A_c = A_c^t = \underline{711\ m/s^2}$

$1\,mm = 10.0\ m/s^2$

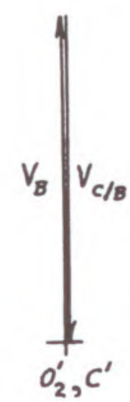
From acceleration polygon $A_{C/B}^t = 0$



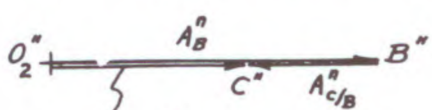
b)



Accelerations are the same as in (a) except A_B^n is changed in direction and A_c^t is changed in magnitude and direction.



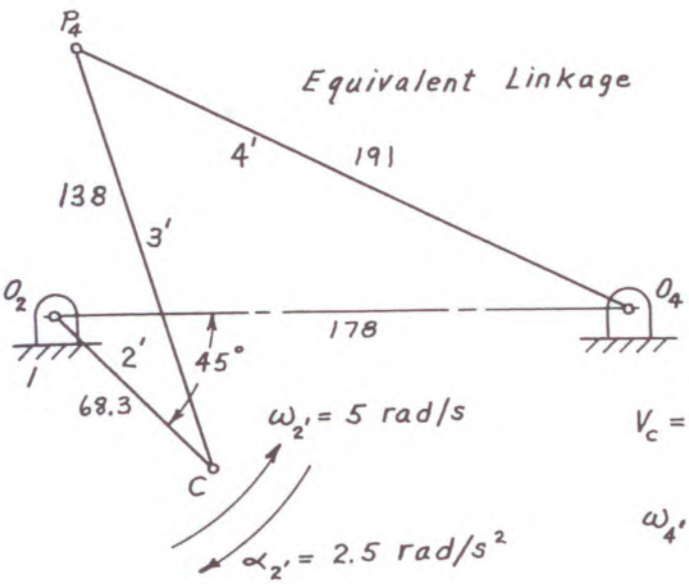
From velocity polygon $\underline{V_c = 0}$

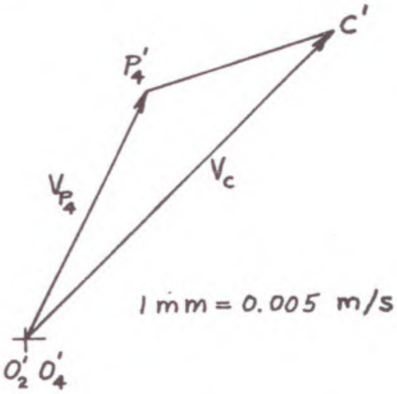


$A_c = A_c^t = \underline{305\ m/s^2}$

From acceleration polygon $A_{C/B}^t = 0$

7-6





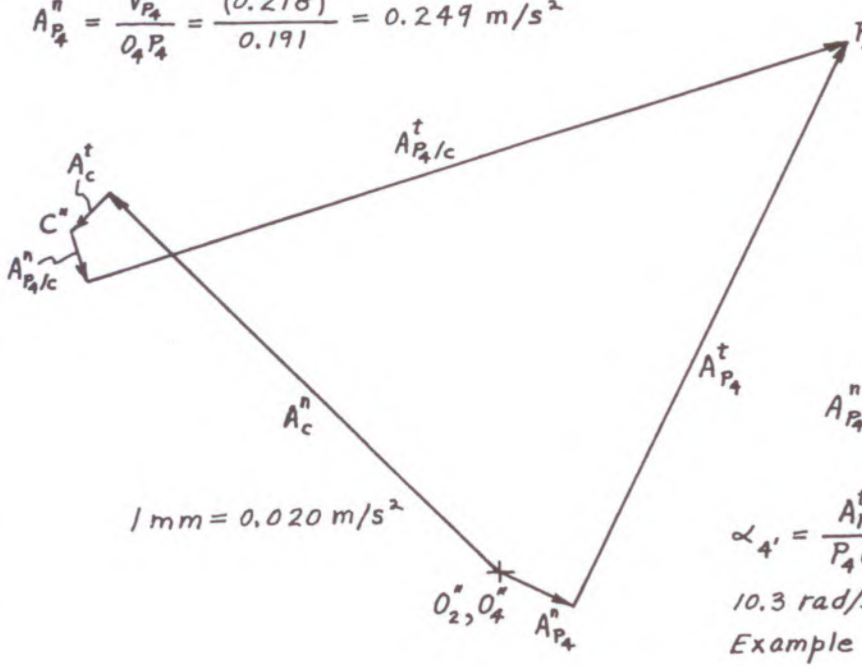
$V_c = (O_2C)\,\omega_{2'} = 0.0683(5) = 0.342$ m/s

$\omega_{4'} = \dfrac{V_{P_4}}{O_4P_4} = \dfrac{0.218}{0.191} = \underline{1.145 \text{ rad/s cw}}$

1.12 rad/s cw was found in Example 7-4.

$$\overset{\checkmark\checkmark}{A^n_{P_4}} + \overset{-\checkmark}{A^t_{P_4}} = \overset{\checkmark\checkmark}{A^n_c} + \overset{\checkmark\checkmark}{A^t_c} + \overset{\checkmark\checkmark}{A^n_{P_4/c}} + \overset{-\checkmark}{A^t_{P_4/c}}$$

$$A^n_{P_4} = \frac{V^2_{P_4}}{O_4P_4} = \frac{(0.218)^2}{0.191} = 0.249 \text{ m/s}^2$$



$$A^n_c = \frac{V_c^2}{O_2C} = \frac{(0.342)^2}{0.0683} = 1.71 \text{ m/s}^2$$

$$A^t_c = (O_2C)\,\alpha_{2'} = (0.0683)\,2.5 = 0.171 \text{ m/s}^2$$

$$A^n_{P_4/c} = \frac{V^2_{P_4/c}}{P_4C} = \frac{(0.152)^2}{0.138} = 0.167 \text{ m/s}^2$$

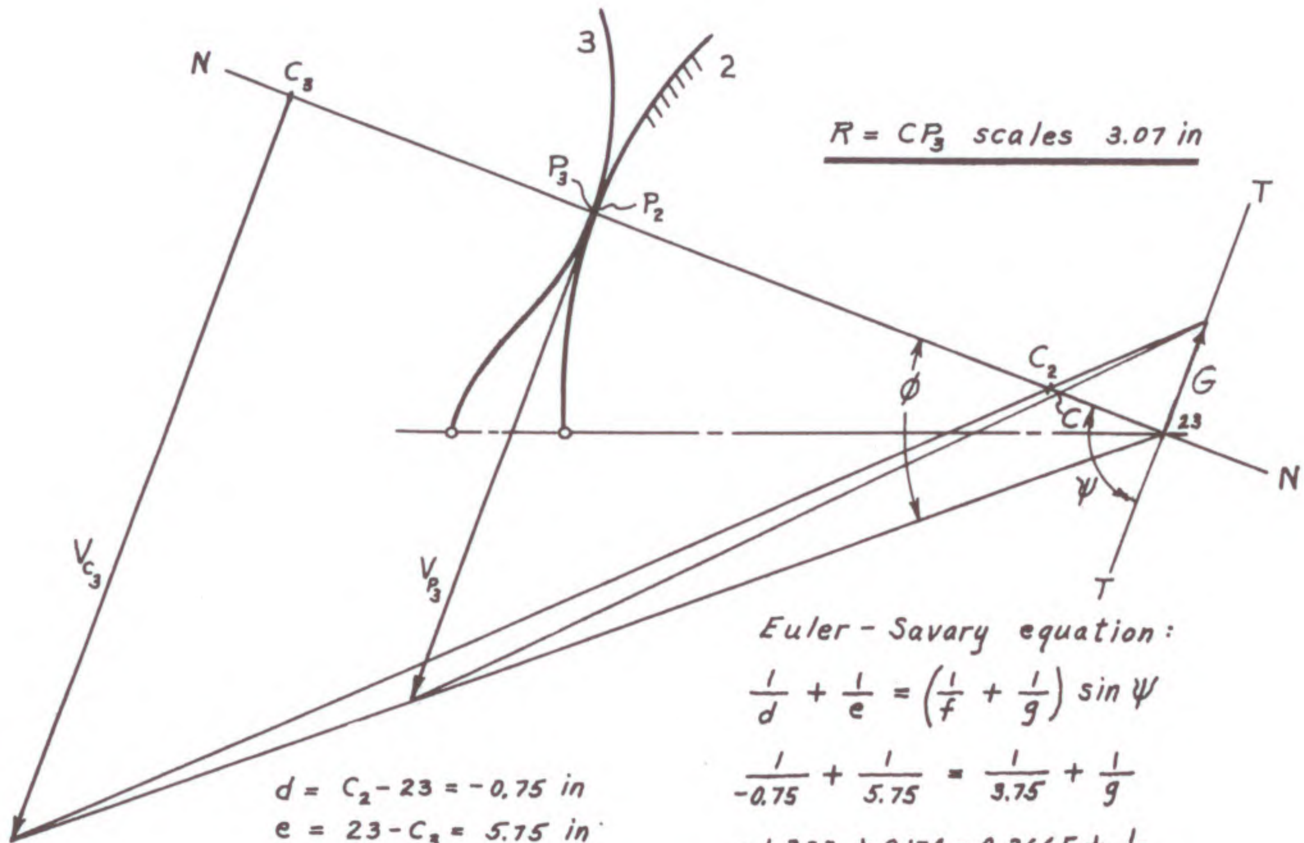
$$\alpha_{4'} = \frac{A^t_{P_4}}{P_4O_4} = \frac{1.96}{0.191} = \underline{10.3 \text{ rad/s}^2 \text{ cw}}$$

10.3 rad/s^2 cw was found in Example 7-4.

7-7

Hartmann's construction:

$R = CP_3$ scales 3.07 in



$d = C_2 - 23 = -0.75$ in
$e = 23 - C_3 = 5.75$ in
$f = 23 - P_3 = 3.75$ in
$g = C - 23$
$\psi = 90°$

Euler - Savary equation:

$$\frac{1}{d} + \frac{1}{e} = \left(\frac{1}{f} + \frac{1}{g}\right) \sin\psi$$

$$\frac{1}{-0.75} + \frac{1}{5.75} = \frac{1}{3.75} + \frac{1}{g}$$

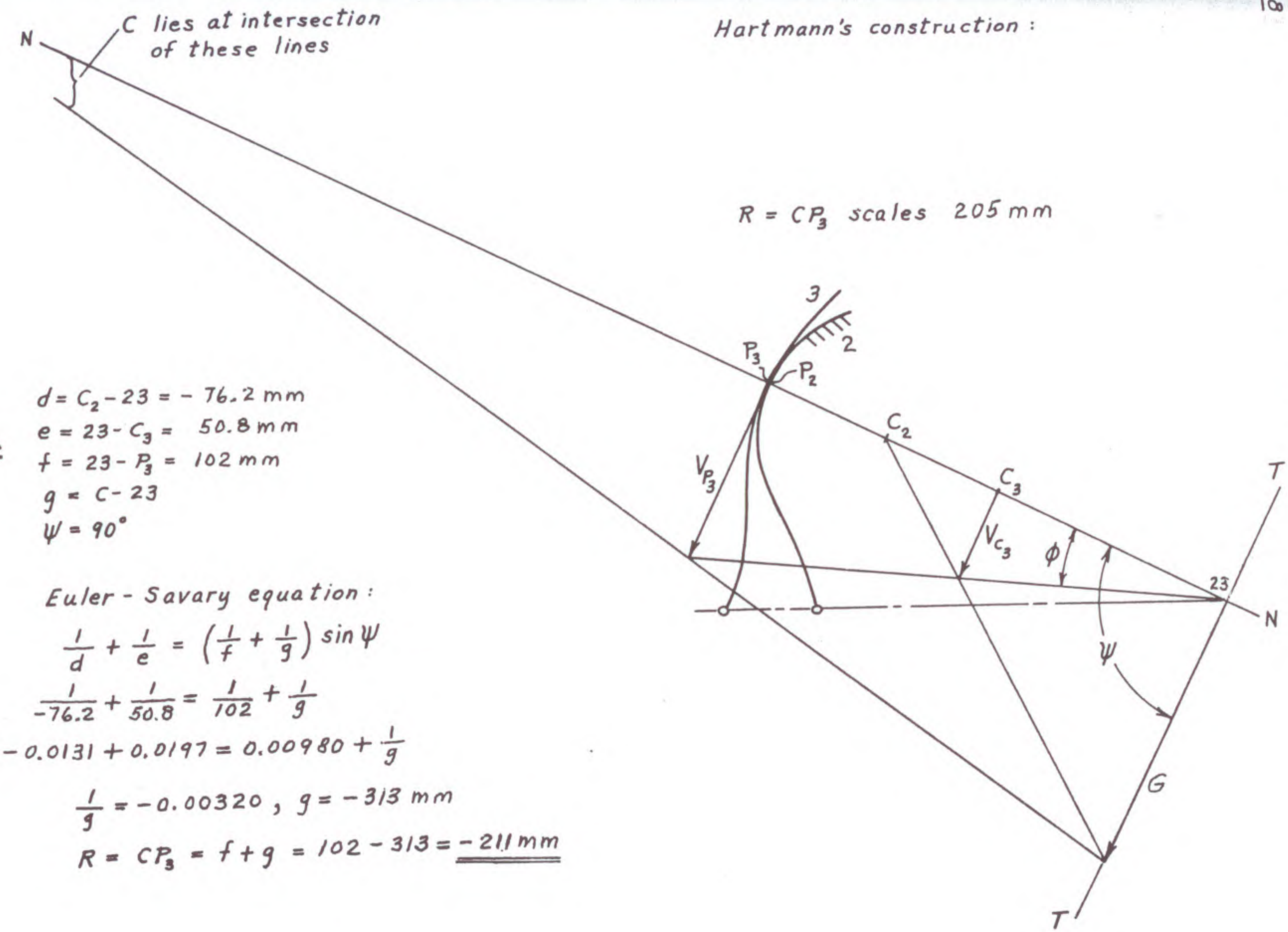
$$-1.333 + 0.174 = 0.2665 + \frac{1}{g}$$

$$\frac{1}{g} = -1.426, \quad g = -0.701$$

$$R = CP_3 = f + g = 3.75 - 0.701 = \underline{3.05 \text{ in}}$$

7-8

Hartmann's construction:



$R = CP_3$ scales 205 mm

$d = C_2 - 23 = -76.2$ mm
$e = 23 - C_3 = 50.8$ mm
$f = 23 - P_3 = 102$ mm
$g = C - 23$
$\psi = 90°$

Euler - Savary equation:

$$\frac{1}{d} + \frac{1}{e} = \left(\frac{1}{f} + \frac{1}{g}\right)\sin\psi$$

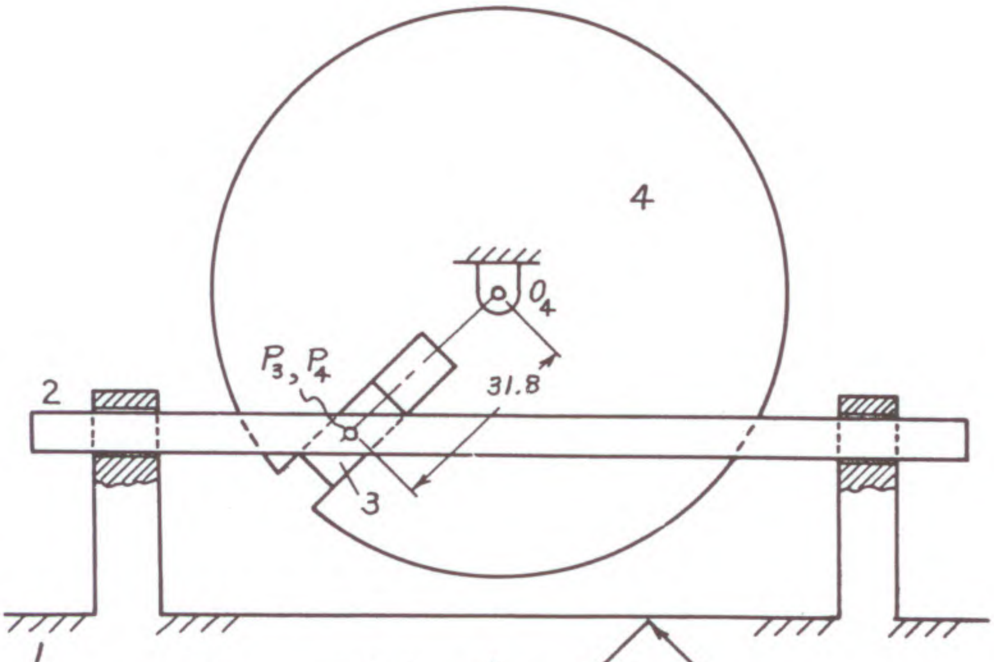
$$\frac{1}{-76.2} + \frac{1}{50.8} = \frac{1}{102} + \frac{1}{g}$$

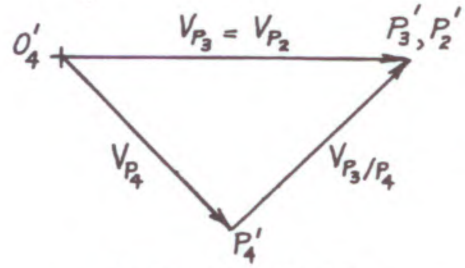
$$-0.0131 + 0.0197 = 0.00980 + \frac{1}{g}$$

$$\frac{1}{g} = -0.00320\ , \quad g = -313 \text{ mm}$$

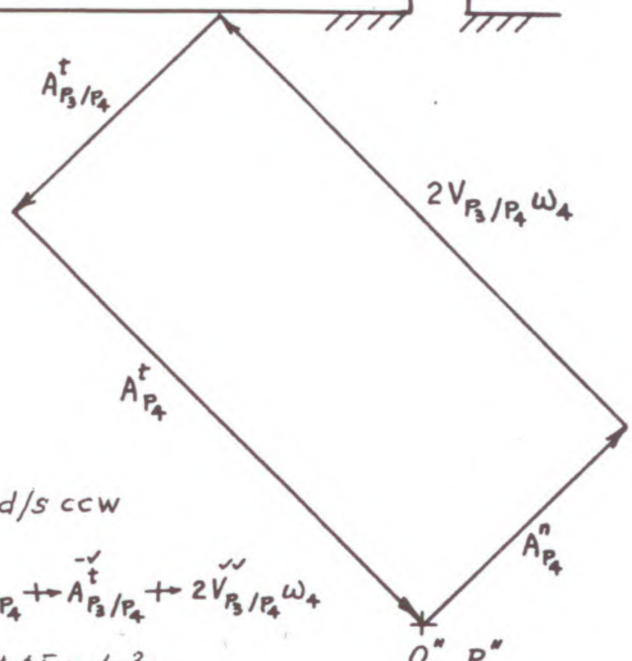
$$R = CP_3 = f + g = 102 - 313 = \underline{\underline{-211 \text{ mm}}}$$

7-12





1mm = 0.001 m/s



1mm = 0.001 m/s^2

$$\overset{-\checkmark}{V_{P_3}} = \overset{-\checkmark}{V_{P_4}} \nrightarrow \overset{\checkmark\checkmark}{V_{P_3/P_4}}$$

$$\omega_4 = \frac{V_{P_4}}{O_4P_4} = \frac{0.0376}{0.0318} = 1.18 \text{ rad/s ccw}$$

$$\overset{o}{A^n_{P_3}} \nrightarrow \overset{o}{A^t_{P_3}} = \overset{\checkmark\checkmark}{A^n_{P_4}} \nrightarrow \overset{-\checkmark}{A^t_{P_4}} \nrightarrow \overset{o}{A^n_{P_3/P_4}} \nrightarrow \overset{-\checkmark}{A^t_{P_3/P_4}} \nrightarrow \overset{\checkmark\checkmark}{2V_{P_3/P_4}\omega_4}$$

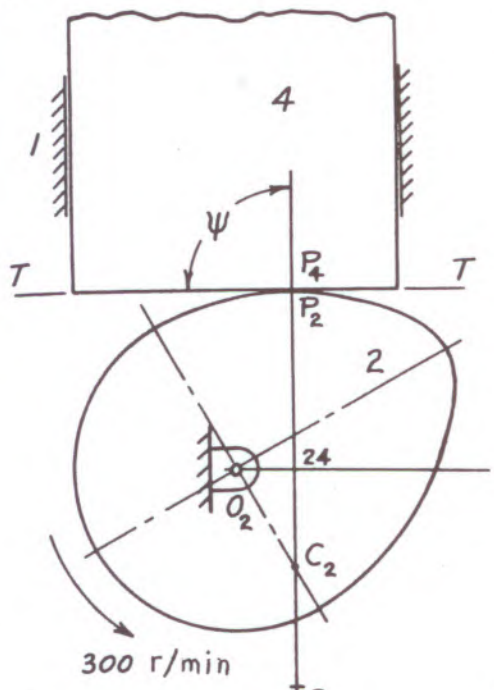
$$A^n_{P_4} = \frac{V^2_{P_4}}{O_4P_4} = \frac{(0.0376)^2}{0.0318} = 0.0445 \text{ m/s}^2$$

$$2V_{P_3/P_4}\omega_4 = 2(0.0381)1.18 = 0.0899 \text{ m/s}^2$$

$$\alpha_4 = \frac{A^t_{P_4}}{O_4P_4} = \frac{0.0899}{0.0318} = \underline{\underline{2.83 \text{ rad/s}^2 \text{ ccw}}}$$

7-15

a)



$$\frac{1}{d} + \frac{1}{e} = \left(\frac{1}{f} + \frac{1}{g}\right)\sin\psi$$

where $d = C_2 - 24 = 15.2$ mm

$e = 24 - C_4 = \infty$

$f = 24 - P_4 = 27.6$ mm

$\psi = 90°$

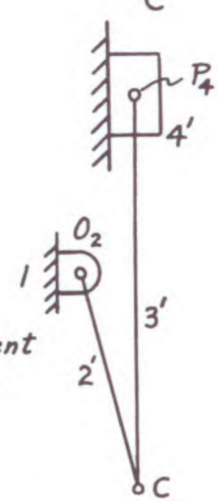
$$\frac{1}{15.2} + \frac{1}{\infty} = \left(\frac{1}{27.6} + \frac{1}{g}\right)\sin 90°$$

$$0.0658 + 0 = 0.0362 + \frac{1}{g}, \quad \frac{1}{g} = 0.0296$$

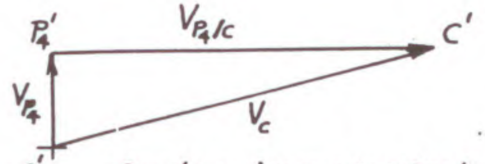
$g = C - 24 = 33.78$ mm

$R = f + g = 27.6 + 33.78 =$ __61.4 mm radius of curvature__

b)



Equivalent Linkage



Scale: 1 mm = 0.018 m/s

Velocities:

$\omega_{2'} = 2\pi \frac{300}{60} = 31.41$ rad/s

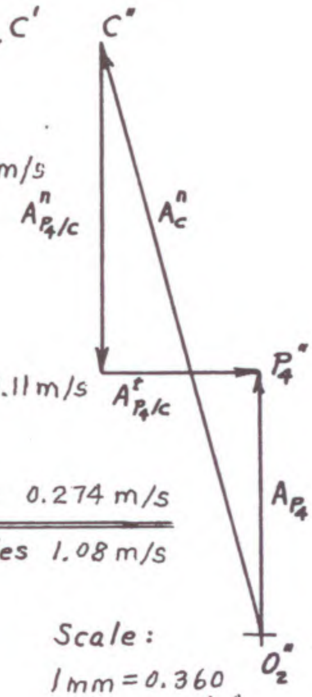
O_2C scales 35.3 mm

$V_C = (O_2C)\omega_{2'} = 0.0353(31.41) = 1.11$ m/s

$$\overset{-\checkmark}{\bar{V}_{P_4}} = \overset{\checkmark\checkmark}{\bar{V}_C} +\!\!\!\rightarrow \overset{-\checkmark}{\bar{V}_{P_4/C}}$$

From vel. polygon __V_{P_4} scales 0.274 m/s__

$V_{P_4/C}$ scales 1.08 m/s



Scale: 1 mm = 0.360 m/s²

Accelerations:

$$\overset{o}{A^n_{P_4}} +\!\!\!\rightarrow \overset{-\checkmark}{A^t_{P_4}} = \overset{\checkmark\checkmark}{A^n_C} +\!\!\!\rightarrow \overset{o}{A^t_C} +\!\!\!\rightarrow \overset{\checkmark\checkmark}{A^n_{P_4/C}} +\!\!\!\rightarrow \overset{-\checkmark}{A^t_{P_4/C}}$$

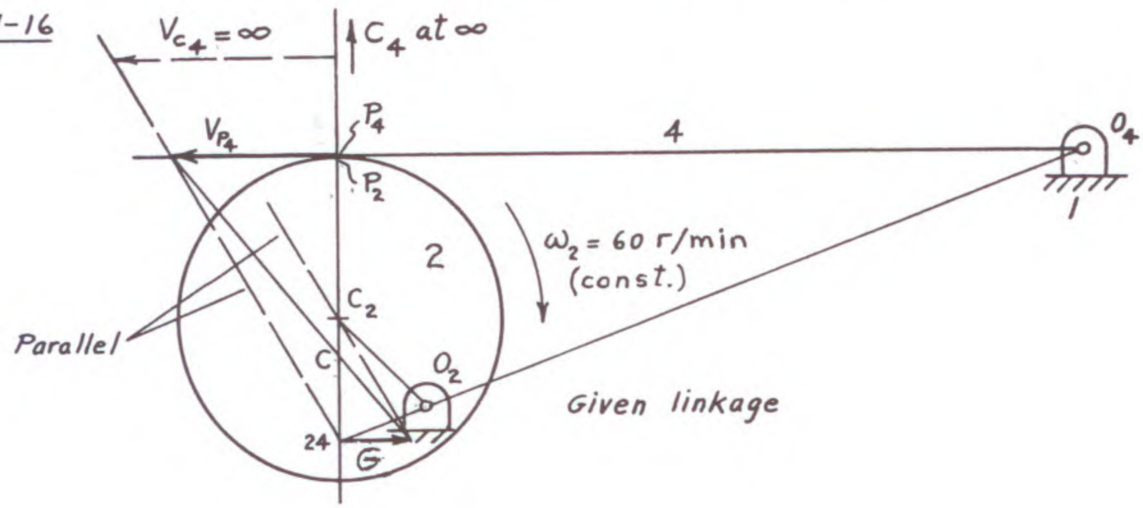
$$A^n_C = \frac{V_C^2}{O_2C} = \frac{1.11^2}{0.0353} = 34.9 \text{ m/s}^2$$

$$A^n_{P_4/C} = \frac{V^2_{P_4/C}}{CP_4} = \frac{1.08^2}{0.0617} = 18.9 \text{ m/s}^2$$

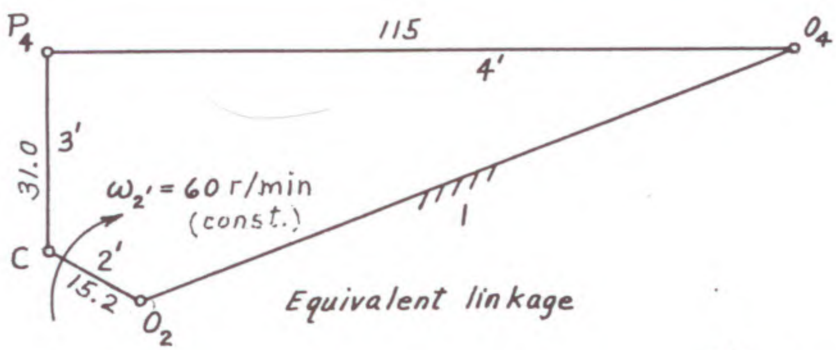
From accel. polygon

__A_{P_4} scales 14.7 m/s²__

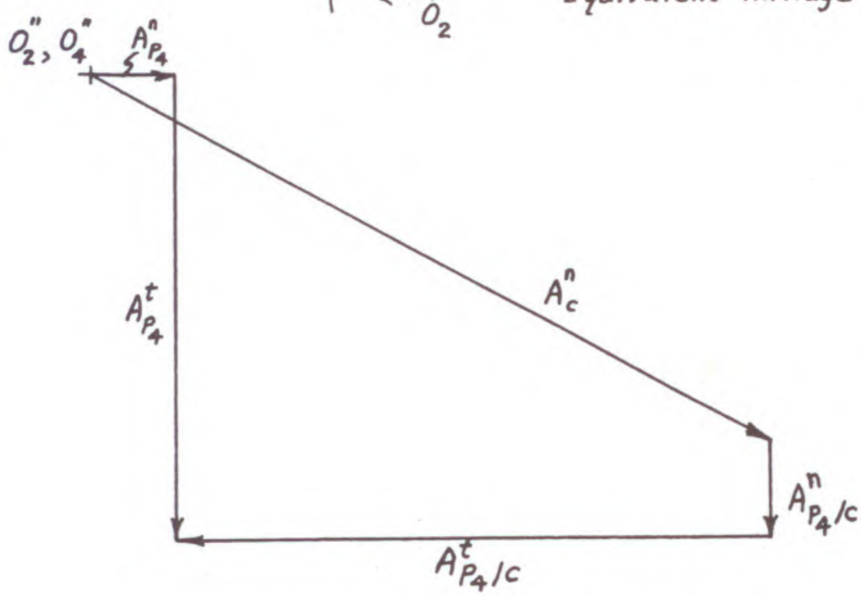
7-16



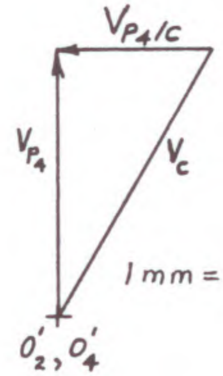
Given linkage



Equivalent linkage



$1\,mm = 0.005\,m/s^2$



$1\,mm = 0.002\,m/s$

7-16 (CONT.)

a) In given linkage

$$\frac{1}{d} + \frac{1}{e} = \left(\frac{1}{f} + \frac{1}{g}\right)\sin\psi$$

$d = C_2\,24 = -0.74''$ (scaled)

$e = 24\,C_4 = \infty$

$f = 24\,P_4 = 1.74''$

$\psi = 90^\circ$

$$\frac{1}{-18.8} + \frac{1}{\infty} = \frac{1}{44.2} + \frac{1}{g}$$

$$-0.0532 + 0 = 0.0226 + \frac{1}{g}$$

$$\frac{1}{g} = -0.0758,\; g = -13.2\text{ mm} = C\,24;\quad R = CP_4 = f + g = 44.2\,(-13.2) = \underline{\underline{31.0\text{ mm}}}$$

Hartmann's construction for locating center of curvature C is shown in the figure.

b) Equivalent linkage is as shown.

c) $\omega_{2'} = 60\,\dfrac{2\pi}{60} = 6.28\text{ rad/s}$

$\alpha_{2'} = 0.\quad V_c = (O_2C)\,\omega_{2'} = 0.0152\,(6.28) = 0.0955\text{ m/s}$

$$\overset{-\checkmark}{V_{P_4}} = \overset{\checkmark\checkmark}{V_c} \mathrel{+\!\!\!\!\rightarrow} \overset{-\checkmark}{V_{P_4/c}}$$

From vel. polygon

V_{P_4} scales 0.0833 m/s

$V_{P_4/c}$ scales 0.0478 m/s

$$\omega_{3'} = \frac{V_{P_4/c}}{CP_4} = \frac{0.0478}{0.0310} = 1.54\text{ rad/s ccw}$$

$$\omega_{4'} = \frac{V_{P_4}}{O_4P_4} = \frac{0.0833}{0.115} = 0.724\text{ rad/s cw}$$

$$\overset{\checkmark\checkmark}{A^n_{P_4}} \mathrel{+\!\!\!\!\rightarrow} \overset{-\checkmark}{A^t_{P_4}} = \overset{\checkmark\checkmark}{A^n_c} \mathrel{+\!\!\!\!\rightarrow} \overset{o}{A^t_c} \mathrel{+\!\!\!\!\rightarrow} \overset{\checkmark\checkmark}{A^n_{P_4/c}} \mathrel{+\!\!\!\!\rightarrow} \overset{-\checkmark}{A^t_{P_4/c}}$$

$$A^n_{P_4} = \frac{V_{P_4}^2}{O_4P_4} = \frac{0.0833^2}{0.115} = 0.0603\text{ m/s}^2$$

$$A^n_c = \frac{V_c^2}{O_2C} = \frac{0.0955^2}{0.0152} = 0.600\text{ m/s}^2$$

$$A^n_{P_4/c} = \frac{V_{P_4/c}^2}{CP_4} = \frac{0.0478^2}{0.0310} = 0.0737\text{ m/s}^2$$

From accel. polygon

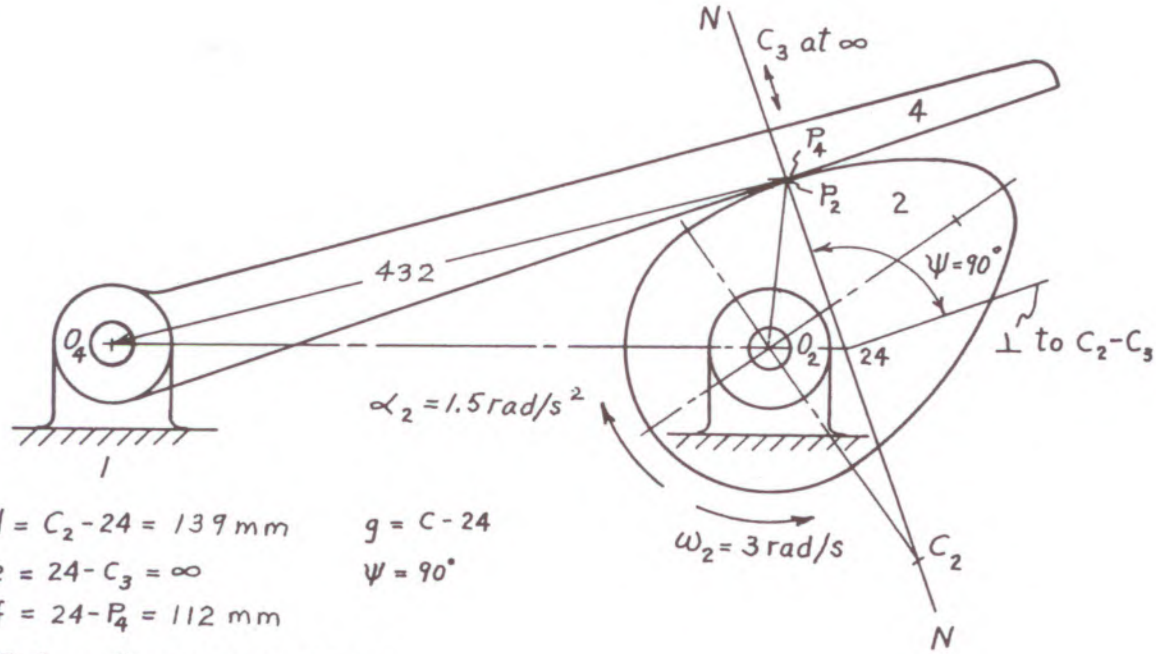
$A^t_{P_4}$ scales 0.368 m/s^2

$$\alpha_{4'} = \frac{A^t_{P_4}}{O_4P_4} = \frac{0.368}{0.115} = 3.20\text{ rad/s}^2$$

$$\omega_4 = \omega_{4'} = \underline{0.724\text{ rad/s cw}}$$

$$\alpha_4 = \alpha_{4'} = \underline{3.20\text{ rad/s}^2\text{ ccw}}$$

7-17



$d = C_2 - 24 = 139\ mm$ $\qquad g = C - 24$

$e = 24 - C_3 = \infty$ $\qquad \psi = 90°$

$f = 24 - P_4 = 112\ mm$

Euler - Savary equation :

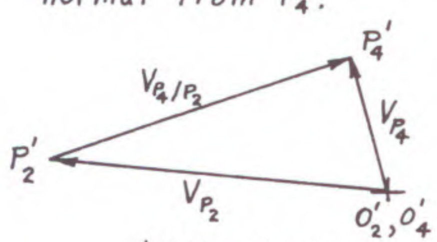
$$\frac{1}{d} + \frac{1}{e} = \left(\frac{1}{f} + \frac{1}{g}\right) \sin\psi$$

$$\frac{1}{139} + \frac{1}{\infty} = \frac{1}{112} + \frac{1}{g}\ ,\quad \frac{1}{g} = -0.00174$$

$g = -575\ mm \qquad R = CP_4 = f + g$

$\qquad = 112 - 575 \quad = \underline{-463\ mm}$

Since R is (−), C lies upward along the normal from P_4.



$V_{P_2} = (O_2P_2)\,\omega_2 = (0.106)3 = 0.317\ m/s$

1 mm = 0.006 m/s

$$\omega_4 = \frac{V_{P_4}}{O_4P_4} = \frac{0.131}{0.432} = \underline{\underline{0.303\ rad/s\ ccw}}$$

7-17 (CONT.)

$$A^n_{P_4} \rightarrow A^t_{P_4} = A^n_{P_2} \rightarrow A^t_{P_2} \rightarrow A^n_{P_4/P_2} \rightarrow A^t_{P_4/P_2} \rightarrow 2V_{P_4/P_2}\,\omega_2$$

$$A^n_{P_4} = \frac{V^2_{P_4}}{O_4P_4} = \frac{(0.131)^2}{0.432} = 0.0397\ m/s^2$$

$$A^n_{P_2} = \frac{V^2_{P_2}}{O_2P_2} = \frac{(0.317)^2}{0.106} = 0.948\ m/s^2$$

$$A^t_{P_2} = (O_2P_2)\,\alpha_2 = (0.106)1.5 = 0.159\ m/s^2$$

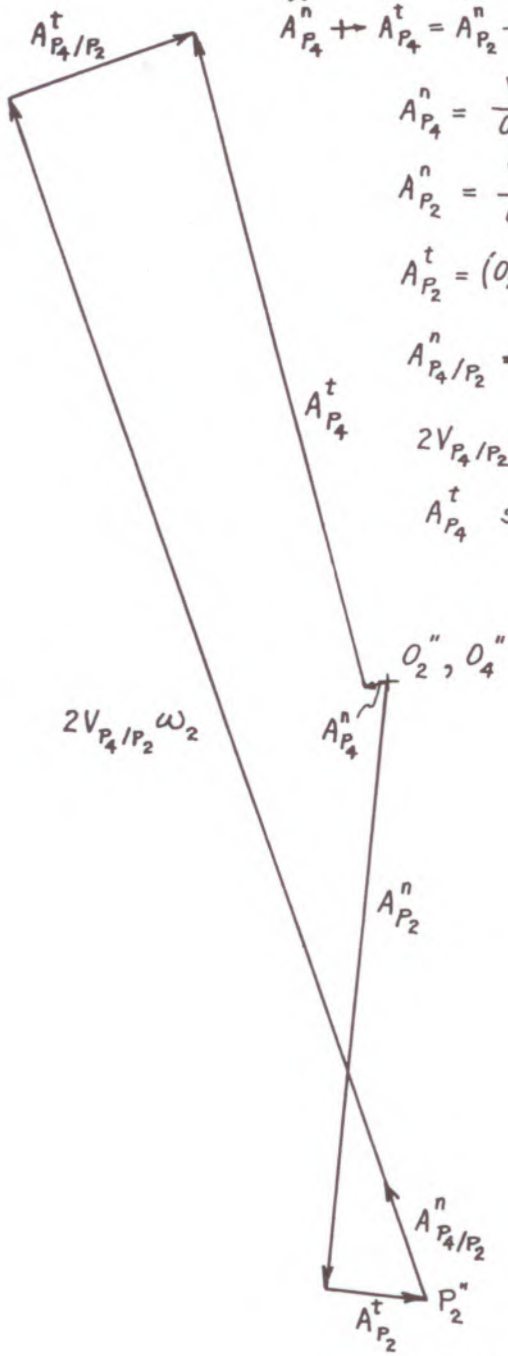
$$A^n_{P_4/P_2} = \frac{V^2_{P_4/P_2}}{R} = \frac{(0.302)^2}{0.463} = 0.197\ m/s^2$$

$$2V_{P_4/P_2}\,\omega_2 = 2(0.302)3 = 1.81\ m/s^2$$

$A^t_{P_4}$ scales $1.05\ m/s^2$

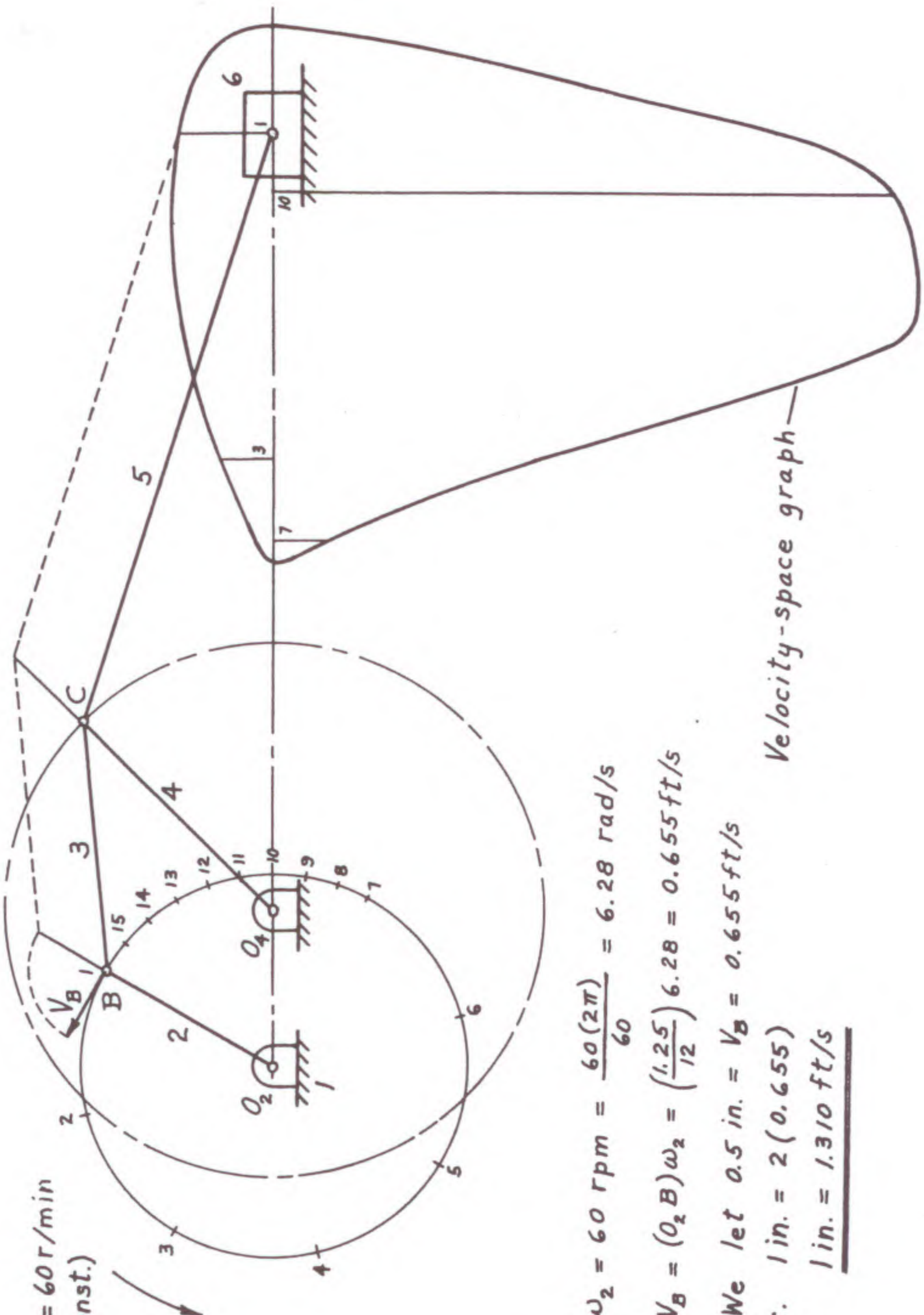
$$\alpha_4 = \frac{A^t_{P_4}}{O_4P_4} = \frac{1.05}{0.432}$$

$$= \underline{\underline{2.43\ rad/s^2\ ccw}}$$



$1\ mm = 0.010\ m/s^2$

8-1



$$\omega_2 = 60 \text{ rpm} = \frac{60(2\pi)}{60} = 6.28 \text{ rad/s}$$

$$V_B = (O_2B)\omega_2 = \left(\frac{1.25}{12}\right)6.28 = 0.655 \text{ ft/s}$$

We let $0.5 \text{ in.} = V_B = 0.655 \text{ ft/s}$

$\therefore\ 1 \text{ in.} = 2(0.655)$

$\underline{1 \text{ in.} = 1.310 \text{ ft/s}}$

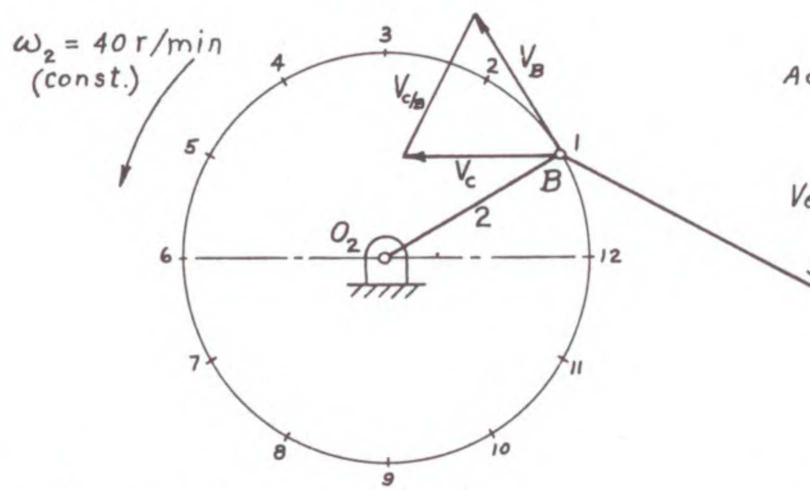
8-2

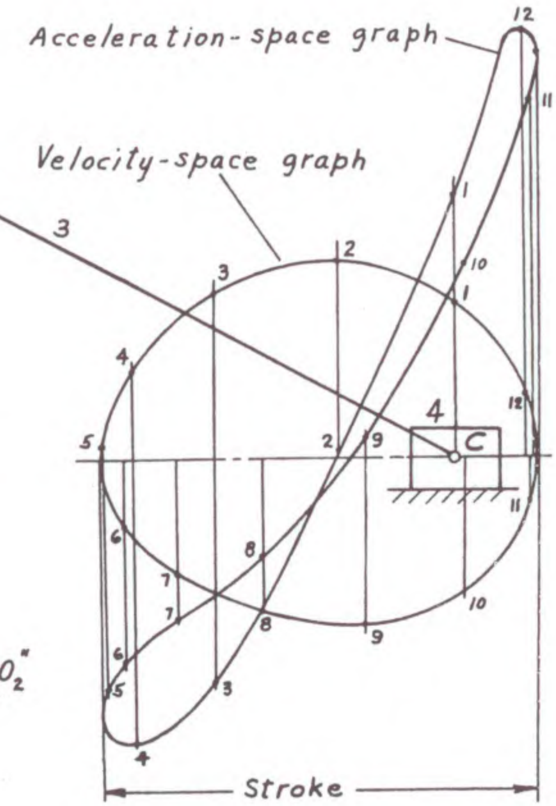
$\omega_B = 40\ r/min = \frac{40(2\pi)}{60} = 4.19\ rad/s$

$V_B = (O_2B)\omega_2 = (0.0318)4.19 = 0.133\ m/s$

We let $25.4\ mm = V_B = 0.133\ m/s$

$\therefore$ Velocity scale is $1\ mm = 0.00524\ m/s$



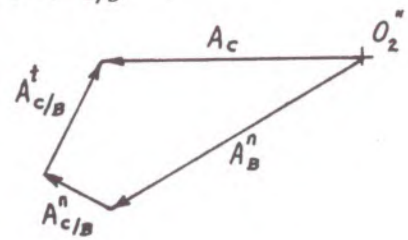


$$\overset{\circ}{A^n_c} \nrightarrow \overset{-\checkmark}{A^t_c} = \overset{\checkmark\checkmark}{A^n_B} \nrightarrow \overset{\circ}{A^t_B} \nrightarrow \overset{\checkmark\checkmark}{A^n_{c/B}} \nrightarrow \overset{-\checkmark}{A^t_{c/B}}$$

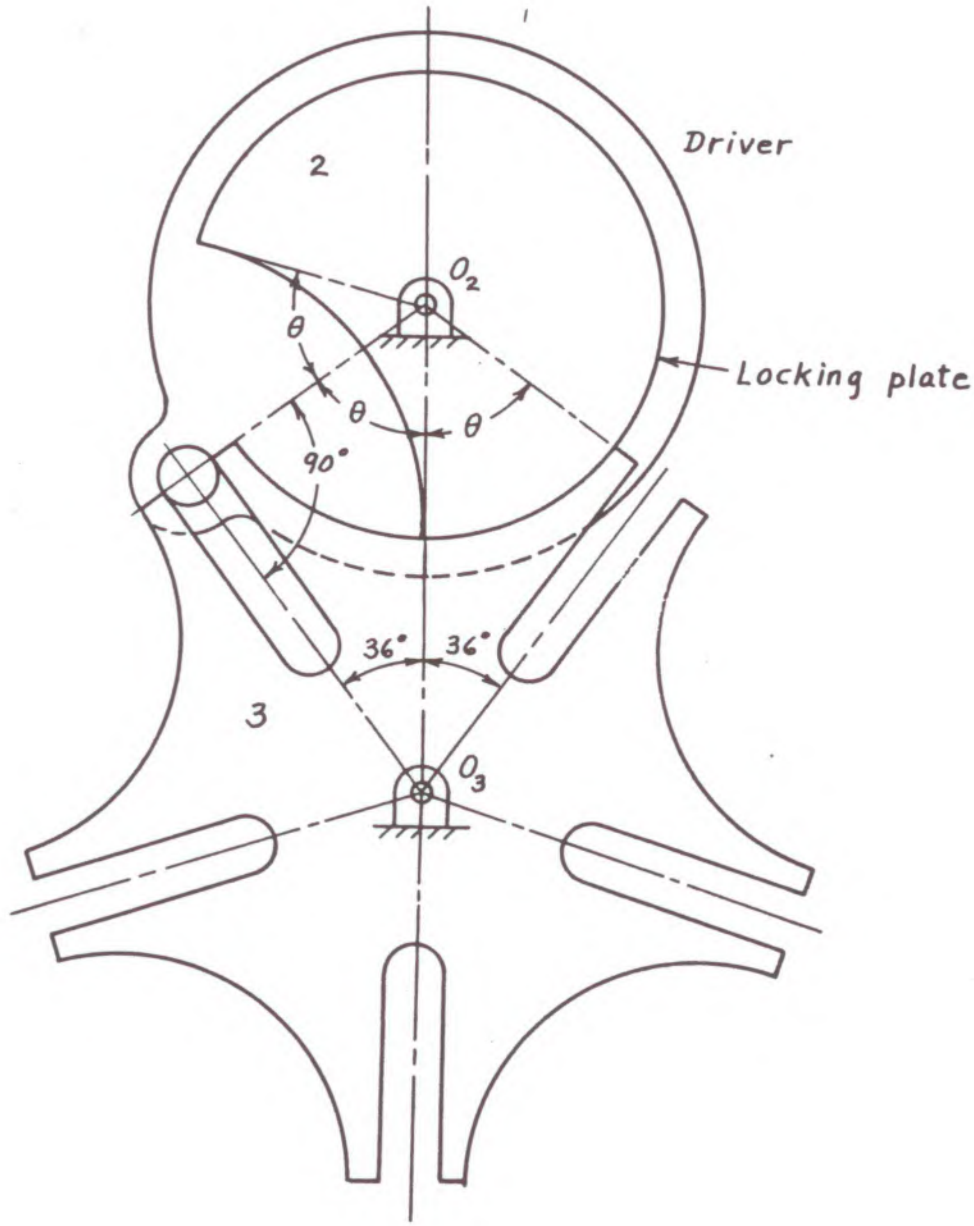
$$A^n_B = \frac{V_B^2}{O_2B} = \frac{(0.133)^2}{0.0318} = 0.556\ m/s^2$$

$$A^n_{c/B} = \frac{V_{c/B}^2}{BC} = \frac{V_{c/B}^2}{114} = 0.00877\,V_{c/B}^2$$

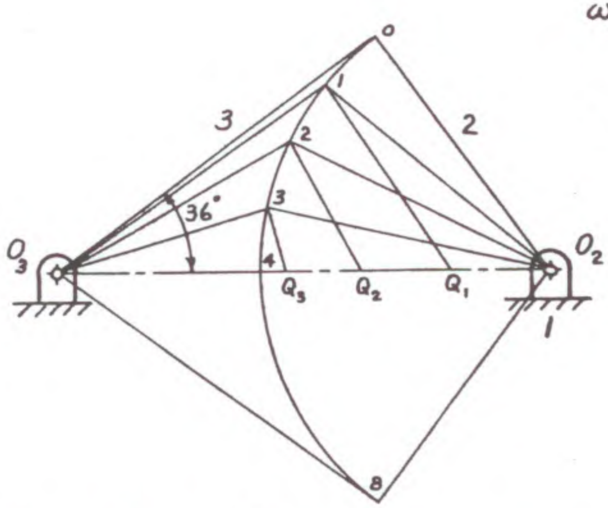
Acceleration polygon
for crank position 1.
$1\ mm = 0.0120\ m/s^2$



8-3



8-3 (CONT.)

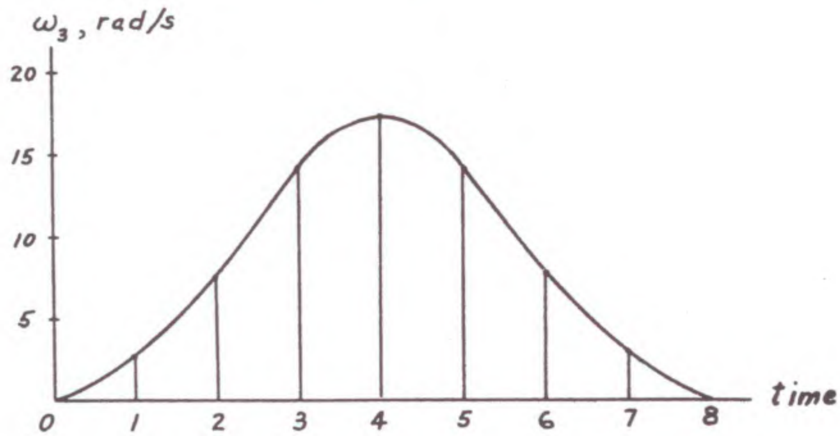


$$\omega_2 = 120 \text{ r/min} = \frac{120(2\pi)}{60} = 12.6 \text{ rad/s}$$

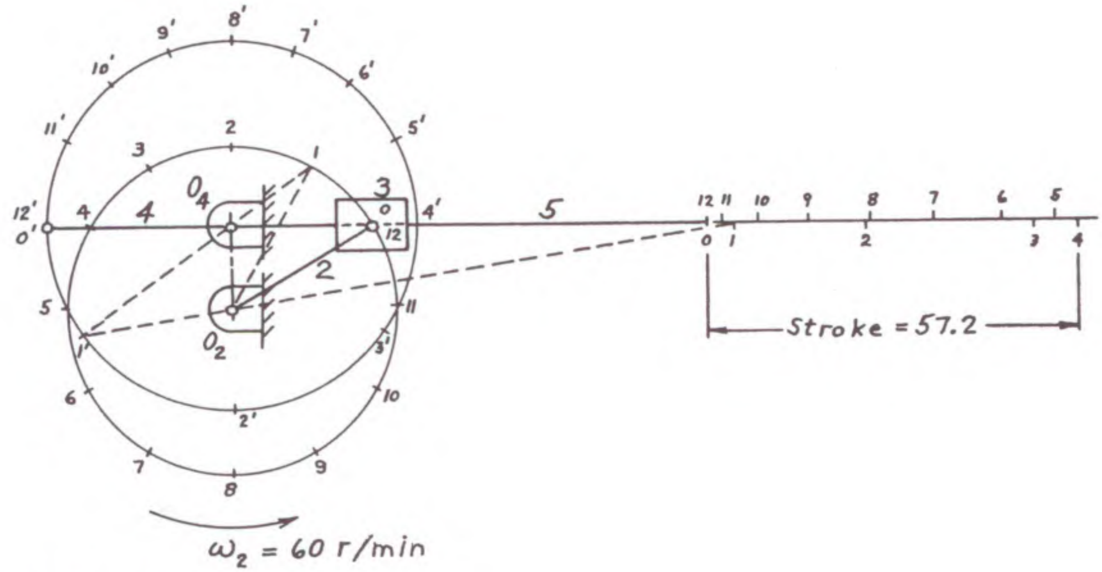
$$\frac{\omega_3}{\omega_2} = \frac{Q_2Q}{Q_3Q}$$

$$\omega_3 = \frac{O_2Q}{O_3Q}\,\omega_2$$

Position	O_2Q	O_3Q	$\frac{O_2Q}{O_3Q}$	ω_3, rad/sec
0	0	76.2	0	0
1	15.0	61.2	0.245	3.09
2	29.0	47.2	0.614	7.74
3	40.4	35.8	1.13	14.2
4	44.2	32.0	1.38	17.4



8-4



Scale for time axis:

Time for 1 rev. of crank 2 $= \frac{1}{60}$ min $= 1$ s

102 mm on time axis $= 1$ s

$\therefore$ 1 mm $= 0.00980$ s

Scale for displacement axis:

1 mm of ordinate = 2 mm of displacement of point D

$\therefore$ 1 mm = 2 mm

Scale for velocity axis:

$\Delta t = 0.25$ s $\quad \Delta s = 39$ mm $\quad \frac{\Delta s}{\Delta t} = \frac{0.039}{0.25} = 0.156$ m/s

A' ordinate measures 20.3 mm; 20.3 mm = 0.156 m/s

1 mm $= \frac{0.156}{20.3} = 0.00768$ $\quad \therefore$ 1 mm = 0.00768 m/s

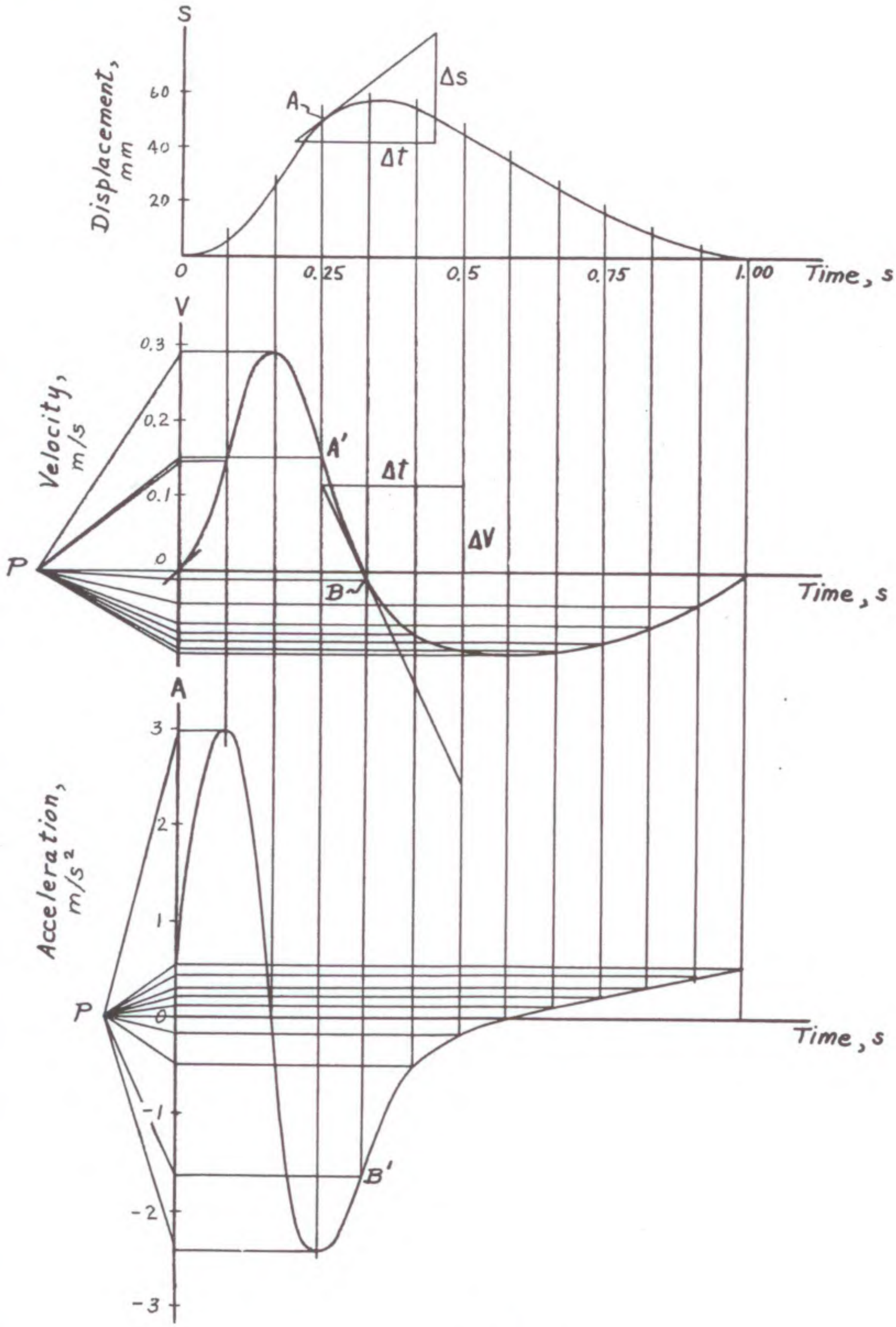
Scale for acceleration axis:

$\Delta t = 0.25$ s $\quad \Delta V = 0.417$ m/s $\quad \frac{\Delta V}{\Delta t} = \frac{0.417}{0.25} = 1.67$ m/s^2

B' ordinate measures 29 mm; 29 mm = 1.67 m/s^2

1 mm $= \frac{1.67}{29} = 0.0578$ $\quad \therefore$ 1 mm = 0.0578 m/s^2

8-4 (CONT.)


S
Displacement, mm
60
40
20
A
Δs
Δt
0
0.25
0.5
0.75
1.00
Time, s
V
Velocity, m/s
0.3
0.2
0.1
0
P
A'
Δt
ΔV
B
Time, s
A
Acceleration, m/s²
3
2
1
0
-1
-2
-3
P
B'
Time, s

9-1

$n = \frac{L}{R} = \frac{6}{2} = 3$, $R = \frac{2}{12} = 0.167\ ft$

$\omega_2 = \frac{2\pi(900)}{60} = 94.2\ rad/s$

$R\omega_2 = 0.167(94.2) = 15.7$

$R\omega_2^2 = 0.167(8860) = 1480$

$V_c = -R\omega_2 \left(\sin\theta + \frac{1}{2n}\sin 2\theta\right)$

$\theta = 0°$: $V_c = -15.7(0) = \underline{0}$

$\theta = 45°$: $V_c = -15.7(0.707 + \frac{1}{6}) = \underline{-13.7\ ft/s}$

$\theta = 90°$: $V_c = -15.7(1+0) = \underline{-15.7\ ft/s}$

$\theta = 135°$: $V_c = -15.7\left[0.707 + \frac{1}{6}(-1)\right] = \underline{-8.48\ ft/s}$

$\theta = 180°$: $V_c = -15.7(0+0) = \underline{0}$

$A_c = -R\omega_2^2\left(\cos\theta + \frac{1}{n}\cos 2\theta\right)$

$\theta = 0°$: $A_c = -1480\left(1 + \frac{1}{3}\right) = \underline{-1975\ ft/s^2}$

$\theta = 45°$: $A_c = -1480\left[0.707 + \frac{1}{3}(0)\right] = \underline{-1045\ ft/s^2}$

$\theta = 90°$: $A_c = -1480\left[0 + \frac{1}{3}(-1)\right] = \underline{493\ ft/s^2}$

$\theta = 135°$: $A_c = -1480\left[-0.707 + \frac{1}{3}(0)\right] = \underline{1045\ ft/s^2}$

$\theta = 180°$: $A_c = -1480\left[-1 + \frac{1}{3}(1)\right] = \underline{986\ ft/s^2}$

9-2

$n = \frac{L}{R} = \frac{6}{2} = 3$; $m = \frac{d}{L} = \frac{2}{6} = 0.333$

$R = \frac{2}{12} = 0.167\ ft$; $\theta = 90°$

$\omega_2 = \frac{2\pi(900)}{60} = 94.2\ rad/s$

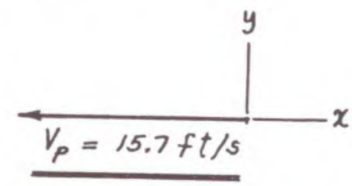
$R\omega_2 = 0.167(94.2) = 15.7$; $R\omega_2^2 = 0.167(8860) = 1480$

$V_P^x = -R\omega_2\left(\sin\theta + \frac{m}{2n}\sin 2\theta\right)$

$= -15.7\left[1 + \frac{0.333}{6}(0)\right] = -15.7\ ft/s$

$V_P^y = R\omega_2(1-m)\cos\theta = 15.7(1-0.333)0 = 0$

9-2 (CONT.)



$A_P^x = -R\omega_2^2\left(\cos\theta + \frac{m}{n}\cos 2\theta\right)$

$= -1480\left[0 + \frac{0.333}{3}(-1)\right] = 164.3\ ft/s^2$

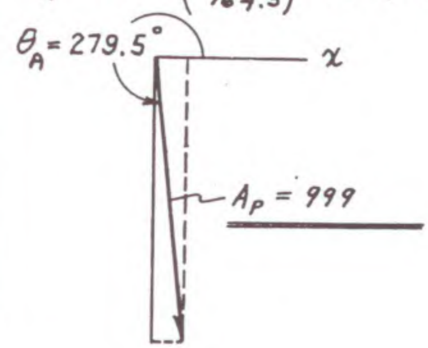
$A_P^y = -R\omega_2^2(1-m)\sin\theta$

$= -1480(1-0.333)1 = -986\ ft/s^2$

$A_P = \sqrt{(A_P^x)^2 + (A_P^y)^2}$

$= \sqrt{(164.3)^2 + (986)^2} = 999\ ft/s^2$

$\theta_A = \tan^{-1}\left(\frac{-986}{164.3}\right) = \tan^{-1}(-6.0) = 279.5\ deg$



9-3

$\omega_2 = \frac{2\pi(3000)}{60} = 314\ rad/s$, $R = 0.0508\ m$

$n = \frac{L}{R} = \frac{152}{50.8} = 3$

$V_c = -R\omega_2\left(\sin\theta + \frac{1}{2n}\sin 2\theta\right)$

$= -R\omega_2\left(\sin\theta + \frac{1}{6}\sin 2\theta\right)$

For V_c max.

$\frac{dV_c}{d\theta} = -R\omega_2\left(\cos\theta + \frac{1}{3}\cos 2\theta\right) = 0$

$\left(\cos\theta + \frac{1}{3}\cos 2\theta\right) = 0$

$\cos\theta = -\frac{1}{3}\cos 2\theta = -\frac{1}{3}(2\cos^2\theta - 1)$

9-3 (CONT.)

$$2\cos^2\theta + 3\cos\theta - 1 = 0$$

$$\cos\theta = \frac{-3 \pm \sqrt{9+8}}{4} = \frac{-3 \pm \sqrt{17}}{4} = \frac{-3 \pm 4.123}{4}$$

$$= 0.281, \quad \underline{\theta = 73.7^\circ \text{ for } V_c \text{ max.}}$$

$$\text{Max. } V_c = -0.0508(314)\left[\sin 73.7^\circ + \tfrac{1}{6}\sin 147.4^\circ\right]$$

$$= -0.0508(314)\left[0.9598 + \tfrac{1}{6}(0.5388)\right]$$

$$= \underline{-16.7\text{ m/s}}$$

$$A_c = -R\omega_2^2\left(\cos\theta + \tfrac{1}{n}\cos 2\theta\right)$$

$$= -R\omega_2^2\left(\cos\theta + \tfrac{1}{3}\cos 2\theta\right)$$

For A_c max.

$$\frac{dA_c}{d\theta} = R\omega_2^2\left(\sin\theta + \tfrac{2}{3}\sin 2\theta\right)$$

$\left(\sin\theta + \frac{2}{3}\sin 2\theta\right) = 0$ when $\theta = 0$

and when $\sin\theta = -\frac{2}{3}\sin 2\theta$

$$= -\tfrac{2}{3}\left(2\sin\theta\cos\theta\right)$$

$$1 = -\tfrac{4}{3}\cos\theta, \quad \cos\theta = -\tfrac{3}{4}, \quad \theta = 138.6^\circ$$

Thus max. values of A_c occur when $\theta = 0^\circ$ and 138.6°

$$\theta = 0^\circ: \quad A_c = -0.0508(314)^2\left(1 + \tfrac{1}{3}\right)$$

$$= \underline{-6680\text{ m/s}^2}$$

$\theta = 138.6^\circ$

$$A_c = -0.0508(314)^2\left[\cos 138.6^\circ + \tfrac{1}{3}\cos 277.2^\circ\right]$$

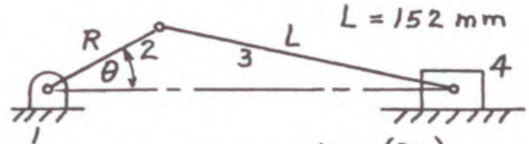
$$= -0.0508(98600)\left[-0.7501 + \tfrac{1}{3}(0.1253)\right]$$

$$= \underline{3550\text{ m/s}^2}$$

9-4

$R = 63.2$ mm $\quad \theta = 30^\circ$

$L = 152$ mm



$$\omega_2 = 1800\text{ r/min} = \frac{1800(2\pi)}{60} = 189\text{ rad/s}$$

$$n = \frac{L}{R} = \frac{152}{63.2} = 2.4$$

$$\omega_3 = \frac{\omega_2(1 - \sin^2\theta)^{\frac{1}{2}}}{(n^2 - \sin^2\theta)^{1/2}}$$

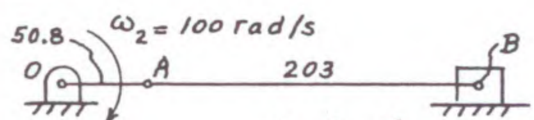
$$= \frac{189\,(1 - 0.25)^{\frac{1}{2}}}{\left[(2.4)^2 - 0.25\right]^{\frac{1}{2}}} = \underline{69.7\text{ rad/s}}$$

$$\alpha_3 = \frac{\omega_2^2 \sin\theta\cos^2\theta}{(n^2 - \sin^2\theta)^{3/2}} - \frac{\omega_2^2\sin\theta}{(n^2 - \sin^2\theta)^{1/2}}$$

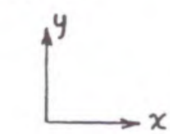
$$= \frac{(189)^2 0.5(0.866)^2}{\left[(2.4)^2 - (0.5)^2\right]^{3/2}} - \frac{(189)^2 0.5}{\left[(2.4)^2 - (0.5)^2\right]^{1/2}}$$

$$= 1040 - 7610 = \underline{-6570\text{ rad/s}^2}$$

9-5



$\theta_2 = \theta_3 = 0^\circ$



Eq. (9-41):

$$\omega_3 = -\frac{b}{c}\,\frac{\cos\theta_2}{\cos\theta_3}\,\omega_2$$

$$= -\frac{50.8}{203}\left(\frac{+1}{+1}\right)(-100) = 25\text{ rad/s}$$

Eq. (9-58):

$$a_B = \mathcal{R}(\bar{a}_B) = -b\left(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2\right) - c\left(\omega_3^2\cos\theta_3 + \alpha_3\sin\theta_3\right)$$

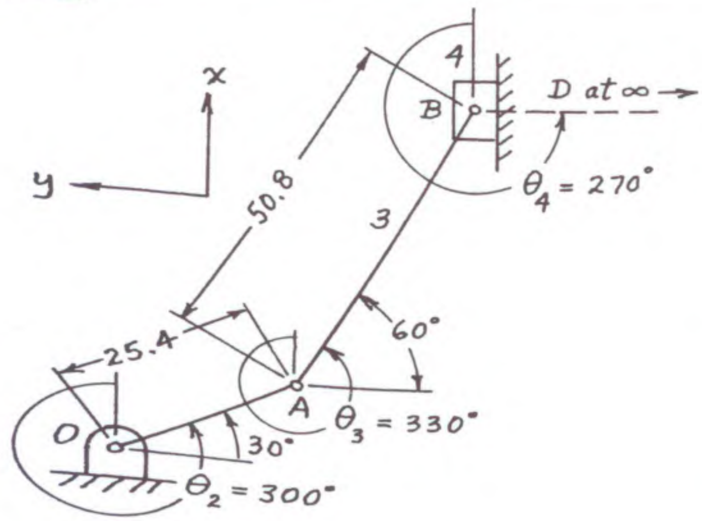
(9-5 CONT.)

$$\mathcal{R}(\bar{a}_B) = -0.0508\left[(100)^2+0\right] - 0.203\left[(25)^2+0\right]$$

$$= -508-127 = \underline{\underline{-635\ \mathrm{m/s^2}}}$$

The minus sign indicates $\bar{a}_B$ is directed to the left.

9-6



$b = 25.4$ mm, $c = 50.8$ mm,

$\omega = 10$ rad/s, $\alpha_2 = 0$

$$\omega_3 = -\frac{b\cos\theta_2}{c\sin\theta_3}\,\omega_2$$

$$= -\frac{25.4\,(0.5)}{50.8\,(0.866)}\,10 = -2.88\ \mathrm{rad/s}$$

$$\alpha_3 = \frac{\omega_3}{\omega_2}\alpha_2 + \frac{b\,\omega_2^2\sin\theta_2 + c\,\omega_3^2\sin\theta_3}{c\cos\theta_3}$$

$$= 0 + \frac{25.4(100)(-0.866)+50.8(8.29)(-0.5)}{50.8\,(0.866)}$$

$$= -54.8\ \mathrm{rad/s^2}$$

(9-6 CONT.)

Eq. (9-57):

$$\mathcal{R}(\bar{v}_B) = -b\omega_2\sin\theta_2 - c\omega_3\sin\theta_3$$

$$= -0.0254(10)(-0.866) - 0.0508(-2.88)(-0.5)$$

$$= 0.220 - 0.0732 = \underline{0.147\ \mathrm{m/s}}$$

i.e. upward

Eq. (9-58):

$$\mathcal{R}(\bar{a}_B) = -b\left(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2\right) - c\left(\omega_3^2\cos\theta_3 + \alpha_3\sin\theta_3\right)$$

$$= -0.0254\left[100(0.5)+0\right] - 0.0508\left[8.29(0.866)-54.8(-0.5)\right]$$

$$= -1.27-1.76 = \underline{-3.03\ \mathrm{m/s^2}}$$

i.e. downward

9-7

First find ω and α for oscillating arm. In Fig. 1 below use same notation as in Fig. 9-13. Must take x-axis in direction of OB.

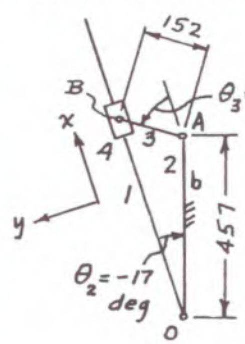


Fig. 1

$\omega_3 = 9.5$ r/min

$= 9.5(2\pi)/60$

$= 0.995$ rad/s

$b = 0.457$ m

$c = 0.152$ m

Eq. (9-60):

$$\omega_1 = \frac{\omega_3}{1+\dfrac{b\cos\theta_2}{c\cos\theta_3}}$$

$$= \frac{0.995}{1+\dfrac{0.457(0.956)}{0.152\,(0.5)}}$$

$= 0.147$ rad/s ccw

Eq (9-61):

$$\alpha_1 = \frac{\alpha_3 - \dfrac{b\,\omega_1^2\sin\theta_2 + c(\omega_3-\omega_1)^2\sin\theta_3}{c\cos\theta_3}}{1+\dfrac{\omega_3-\omega_1}{\omega_1}}$$

9-7 (CONT.)

$$\alpha_1 = \frac{0 - \dfrac{0.457(0.0216)(-.292)+0.152(0.848)^2 0.866}{0.152(0.5)}}{1+\dfrac{0.848}{0.147}}$$

$$= -0.178 \text{ rad/s}^2$$

Next, find v and a for upper slider. In Fig. 2 below use same notation as in Fig. 9-12.

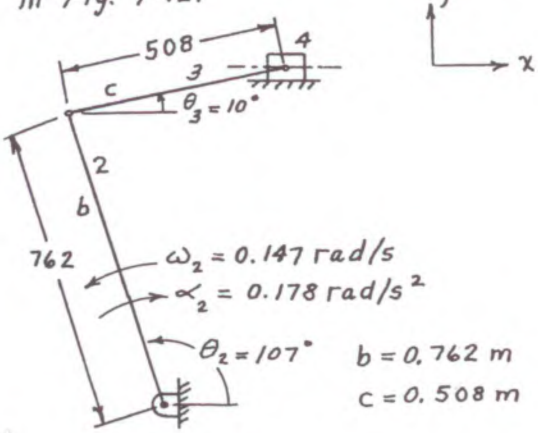


Fig. 2

Eq (9-41):

$$\omega_3 = -\frac{b\cos\theta_2}{c\cos\theta_3}\,\omega_2 = -\frac{0.762(-0.2924)}{0.508(0.9848)}0.147$$

$$= 0.0655 \text{ rad/s ccw}$$

Eq. (9-42):

$$\alpha_3 = \frac{\omega_3}{\omega_2}\alpha_2 + \frac{b\,\omega_2^2\sin\theta_2 + c\,\omega_3^2\sin\theta_3}{c\cos\theta_3}$$

$$= \frac{0.0655}{0.147}(-0.178) + \frac{0.762(0.0216)(0.9563)+0.508(0.00429)(0.1736)}{0.508(0.9848)}$$

$$= -0.0793 + 0.0315 = -0.0478 \text{ rad/s}^2$$

Eq. (9-57):

$$\mathcal{R}(\bar{v}_B) = -b\omega_2\sin\theta_2 - c\omega_3\sin\theta_3$$

$$= -0.762(0.147)\,0.9563 - 0.508(0.0655)\,0.1736$$

$$= -0.107 - 0.00578 = \underline{\underline{-0.113 \text{ m/s}}}$$ vel. of slider 6 Fig. P9-7 (directed to left)

Eq. (9-58):

$$\mathcal{R}(\bar{a}_B) = -b\left(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2\right) - c\left(\omega_3^2\cos\theta_3 + \alpha_3\sin\theta_3\right)$$

$$= -0.762\left[0.0216(-0.2924) + (-0.178)(0.9563)\right]$$

$$= -0.508\left[0.00429(0.9848) + (-0.0478)\,0.1736\right]$$

$$= 0.134 + 0.00207 = \underline{\underline{0.136 \text{ m/s}^2}}$$

accel. of slider 6 in Fig. P9-7 (directed to right)

9-8

$$\omega_2 = -60\,\frac{2\pi}{60} = -6.28 \text{ rad/s}\ ;\ \alpha_2 = 0$$

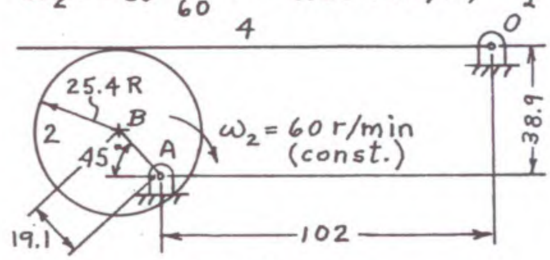


Fig. 1

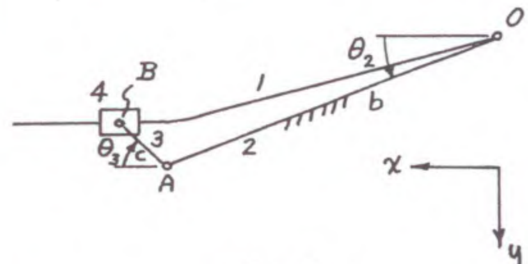


Fig. 2

In the equivalent linkage (Fig. 2) the notation is the same as in Fig. 9-13.

$$b = \sqrt{0.102^2 + 0.0389^2} = 0.109 \text{ m}$$

$$c = 0.0191 \text{ m}$$

$$\theta_2 = \tan^{-1}\left(\frac{38.9}{102}\right) = 20.9°,\quad \theta_3 = -45°$$

$$\omega_3 = -6.28 \text{ rad/s},\quad \alpha_3 = 0$$

Eq. (9-60):

$$\omega_1 = \frac{-6.28}{1+\dfrac{0.109(0.9342)}{0.0191(0.707)}} = -0.735 \text{ rad/s cw}$$

9-8 (CONT.)

Eq. (9-61):

$$\alpha_1 = \frac{-\dfrac{0.109(0.735)^2 0.357 + 0.0191(-6.28+0.735)^2(-0.707)}{0.0191(0.707)}}{1 + \dfrac{-6.28+0.735}{-0.735}}$$

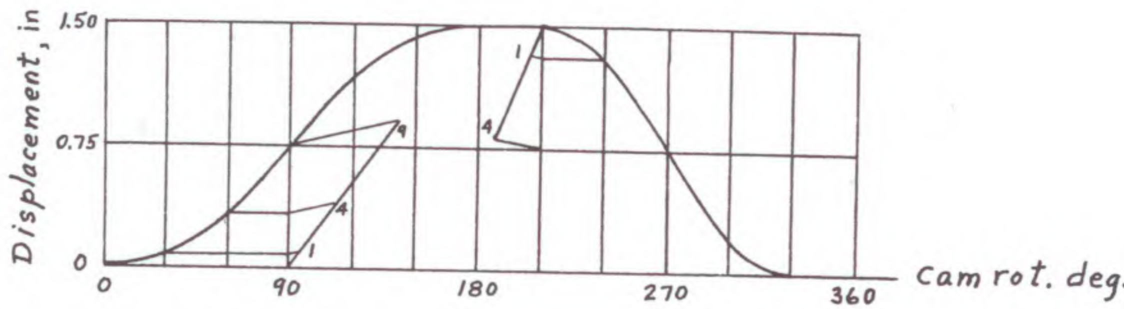
$$= 3.42 \text{ rad/s}^2 \text{ ccw}$$

Thus in Fig. 1 above

$\omega_4 = -0.735$ rad/s cw

$\alpha_4 = 3.42$ rad/s^2 ccw

10-1, 10-5



$$\omega = \frac{300}{60}(2\pi) = 31.42 \text{ rad/s}$$

Rise:

V_{max} is at $\theta = 90° = 90\left(\frac{\pi}{180}\right) = 1.571 \text{ rad}$

$\beta = 180° = 3.141 \text{ rad}$; $h = 1.50 \text{ in} = 0.125 \text{ ft}$

$$V = \frac{4h\omega\theta}{\beta^2}$$

$$V_{max} = \frac{4(0.125)(31.42)1.571}{(3.141)^2} = \underline{\underline{2.50 \text{ ft/s}}}$$

$$A = \frac{4h\omega^2}{\beta^2} = \frac{4(0.125)(31.42)^2}{(3.141)^2}$$

$$= \underline{\underline{50 \text{ ft/s}^2}}$$

Fall:

V_{max} is at $\theta = 60° = 60\left(\frac{\pi}{180}\right) = 1.047 \text{ rad}$

$$\beta = 120° = 120\left(\frac{\pi}{180}\right) = 2.094 \text{ rad}$$

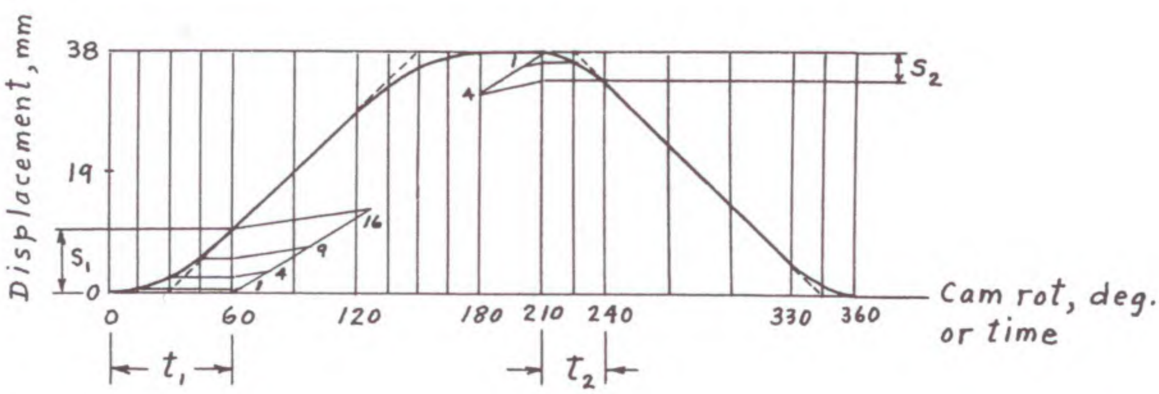
$$V = \frac{4h\omega\theta}{\beta^2}$$

$$V_{max} = \frac{4(0.125)(31.42)1.047}{(2.094)^2} = \underline{\underline{3.73 \text{ ft/s}}}$$

$$A = \frac{4h\omega^2}{\beta^2} = \frac{4(0.125)(31.42)^2}{(2.094)^2}$$

$$= \underline{\underline{113 \text{ ft/s}^2}}$$

10-2, 10-6



$$\omega = \frac{450}{60}(2\pi) = 47.12 \text{ rad/s}$$

Rise:

From similar triangles

$$\frac{s_1}{38} = \frac{30}{120}, \quad s_1 = 38\left(\frac{30}{120}\right) = 9.5 \text{ mm}$$

$$h = 2s_1 = 19 \text{ mm}$$

$$\beta = 2(60) = 120^\circ = 120\left(\frac{\pi}{180}\right) = 2.094 \text{ rad}$$

$$V = \frac{4h\omega\theta}{\beta^2}, \quad V_{max} \text{ is at } \theta = 60^\circ = 1.047 \text{ rad}$$

$$V_{max} = \frac{4(0.019)(47.12)\,1.047}{(2.094)^2}$$

$$= \underline{\underline{0.855 \text{ m/s}}}$$

$$A = \frac{4h\omega^2}{\beta^2} = \frac{4(0.019)(47.12)^2}{(2.094)^2}$$

$$= \underline{\underline{38.49 \text{ m/s}^2}}$$

Fall:

From similar triangles

$$\frac{s_2}{38} = \frac{15}{120}, \quad s_2 = 38\left(\frac{15}{120}\right) = 4.75 \text{ mm}$$

$$h = 2s_2 = 2(4.75) = 9.5 \text{ mm}$$

$$\beta = 2(30) = 60^\circ = 60\left(\frac{\pi}{180}\right) = 1.047 \text{ rad}$$

$$V = \frac{4h\omega\theta}{\beta^2}, \quad V_{max} \text{ is at } \theta = 30^\circ = 0.524 \text{ rad}$$

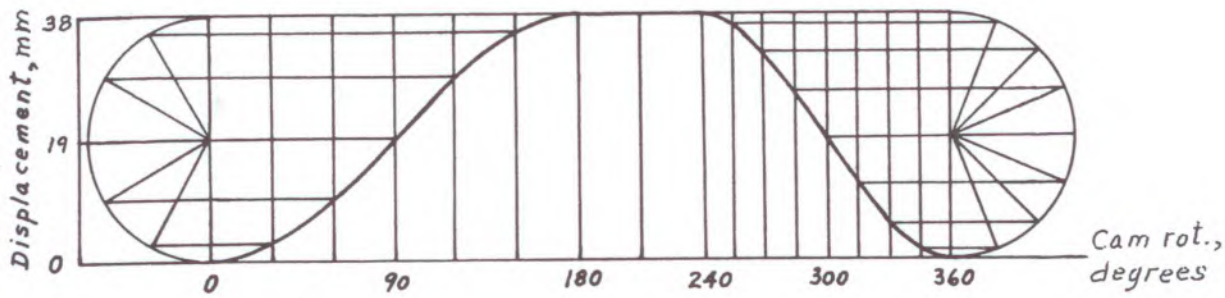
$$V_{max} = \frac{4(0.0095)(47.12)\,0.524}{(1.047)^2}$$

$$= \underline{\underline{0.856 \text{ m/s}}}$$

$$A = \frac{4h\omega^2}{\beta^2} = \frac{4(0.0095)(47.12)^2}{(1.047)^2}$$

$$= \underline{\underline{76.0 \text{ m/s}^2}}$$

10-3, 10-7



$\omega = 200\,r/min = \frac{200}{60}(2\pi) = 20.9\,rad/s$

Rise:

$\beta = 180^\circ = \pi\,rad$

V_{max} occurs at $\theta = 90^\circ$

$$V_{max} = \frac{\pi h \omega}{2\beta} = \frac{\pi(0.038)20.9}{2\pi}$$

$$= \underline{\underline{0.397\,m/s}}$$

A_{max} occurs at 0° and 180°

$$A_{max} = \frac{\pi^2 h \omega^2}{2\beta^2} = \frac{\pi^2(0.038)(20.9)^2}{2(\pi)^2}$$

$$= \underline{\underline{8.30\,m/s^2}}$$

Fall:

$\beta = 360 - 240 = 120^\circ = 120\left(\frac{\pi}{180}\right) = 2.094\,rad$

V_{max} occurs at $\theta = 60^\circ$ of fall.

$$V_{max} = \frac{\pi h \omega}{2\beta} = \frac{\pi(0.038)20.9}{2(2.094)}$$

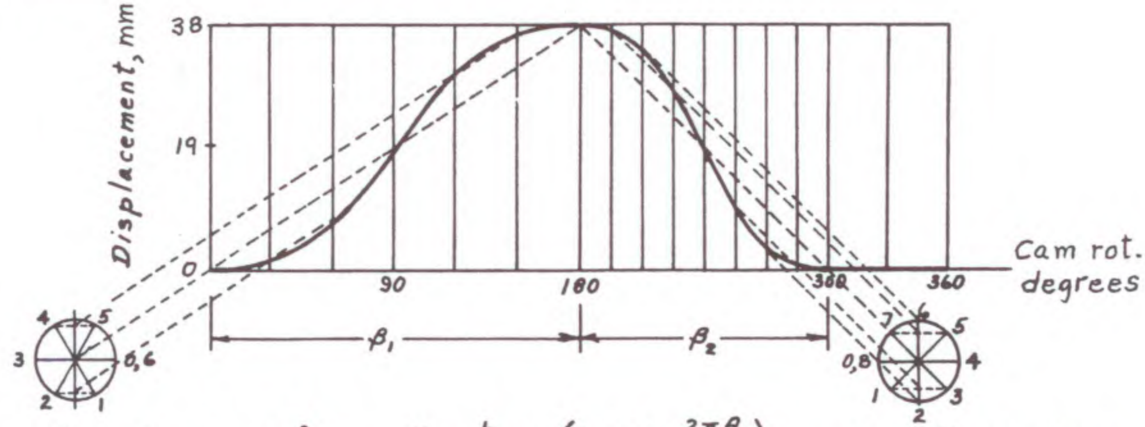
$$= \underline{\underline{0.596\,m/s}}$$

A_{max} occurs at $\theta = 0^\circ$ and 120° of fall.

$$A_{max} = \frac{\pi^2 h \omega^2}{2\beta^2} = \frac{\pi^2(0.038)(20.9)^2}{2(2.094)^2}$$

$$= \underline{\underline{18.68\,m/s^2}}$$

10-4, 10-8



$$s = h\frac{\theta}{\beta} - \frac{h}{2\pi}\sin 2\pi\frac{\theta}{\beta}\ , \quad V = \frac{h}{\beta}\omega\left(1-\cos\frac{2\pi\theta}{\beta}\right)\ , \quad \omega = \frac{240}{60}2\pi = 25.1 \text{ rad/s}$$

$$A = \frac{2\pi h}{\beta^2}\omega^2 \sin\frac{2\pi\theta}{\beta}\ , \quad V_{max} = \frac{2h}{\beta}\omega\ , \quad A_{max} = \frac{2\pi h}{\beta^2}\omega^2$$

Rise:

$$V_{max} = \frac{2(0.038)25.1}{\pi} = \underline{\underline{0.607\ \text{m/s}}}$$

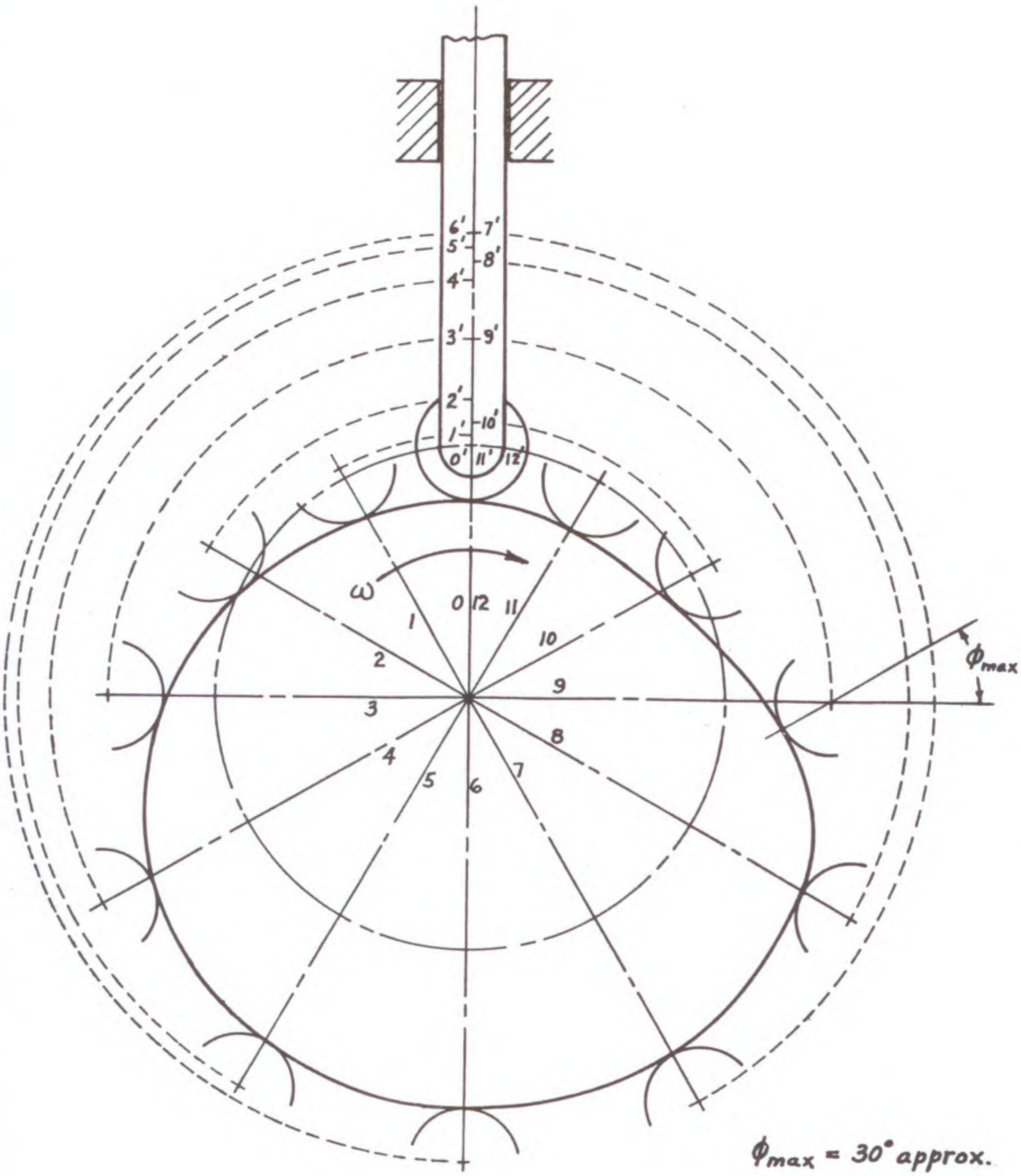
$$A_{max} = \frac{2\pi(0.038)(25.1)^2}{\pi^2} = \underline{\underline{15.2\ \text{m/s}^2}}$$

Fall:

$$V_{max} = \frac{2(0.038)25.1}{\left(\frac{120}{180}\right)\pi} = 0.911\ \text{m/s}$$

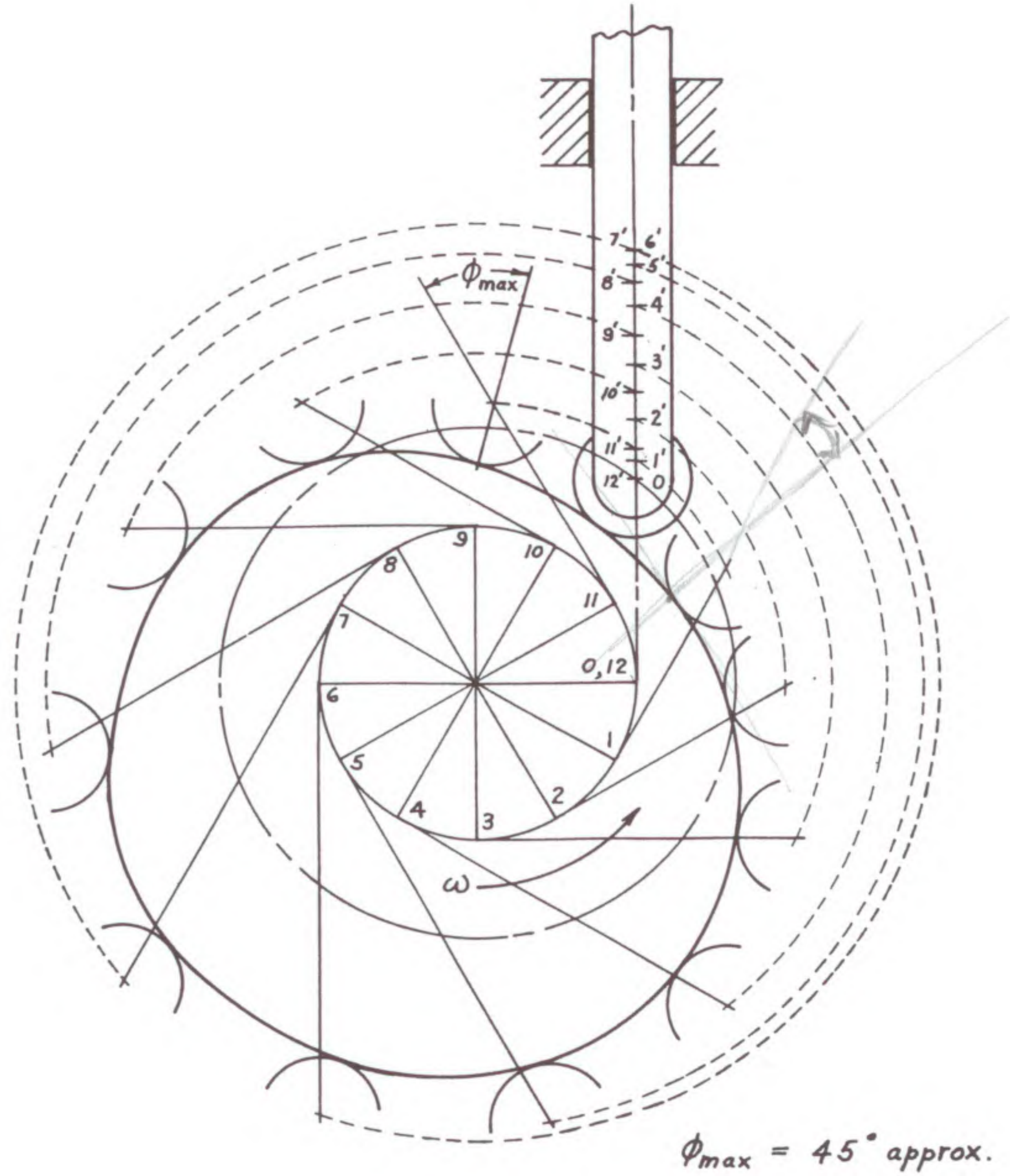
$$A_{max} = \frac{2\pi(0.038)(25.1)^2}{\left(\frac{120}{180}\pi\right)^2} = \underline{\underline{34.3\ \text{m/s}^2}}$$

10-9



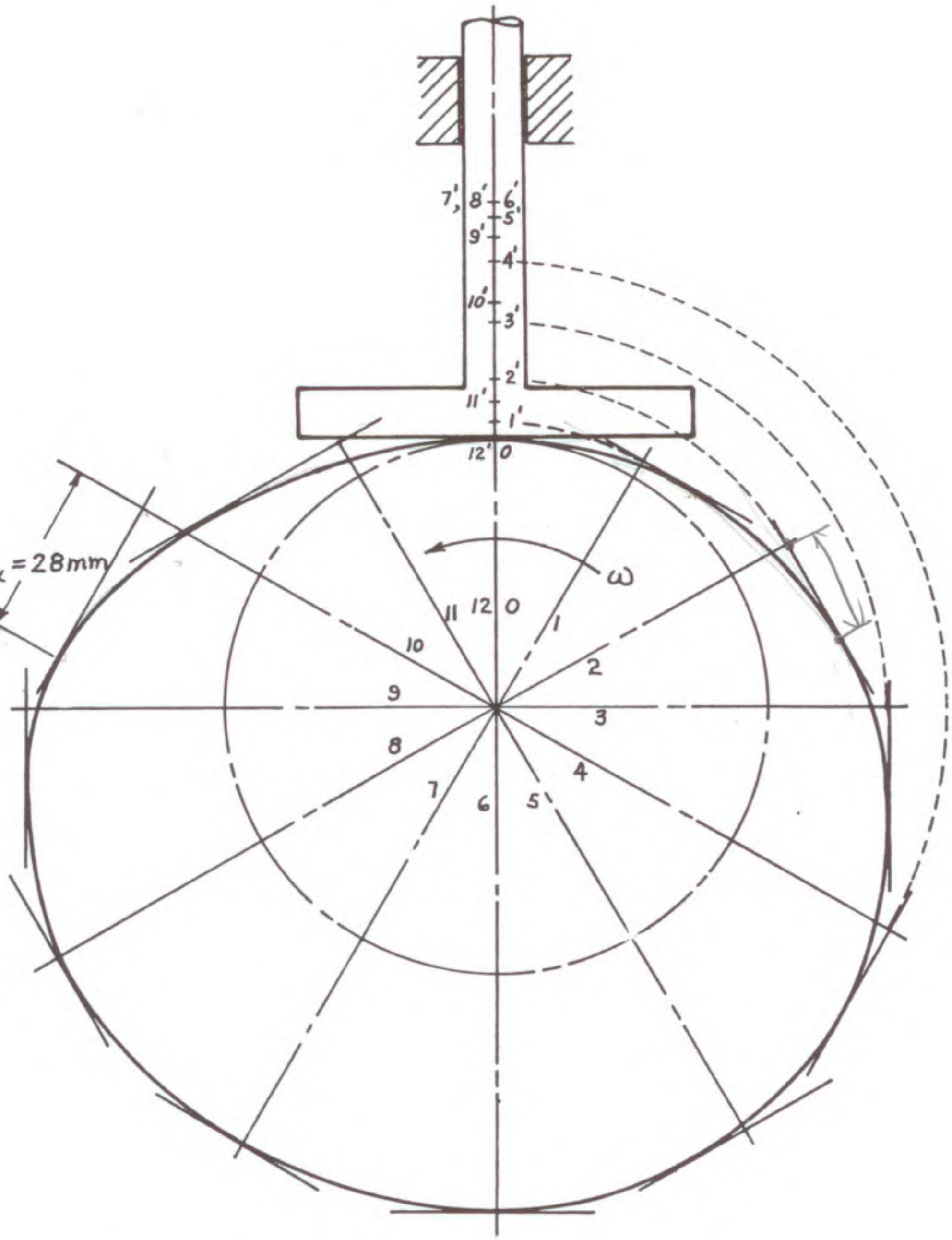
ϕ_{max} = 30° approx.

10-10

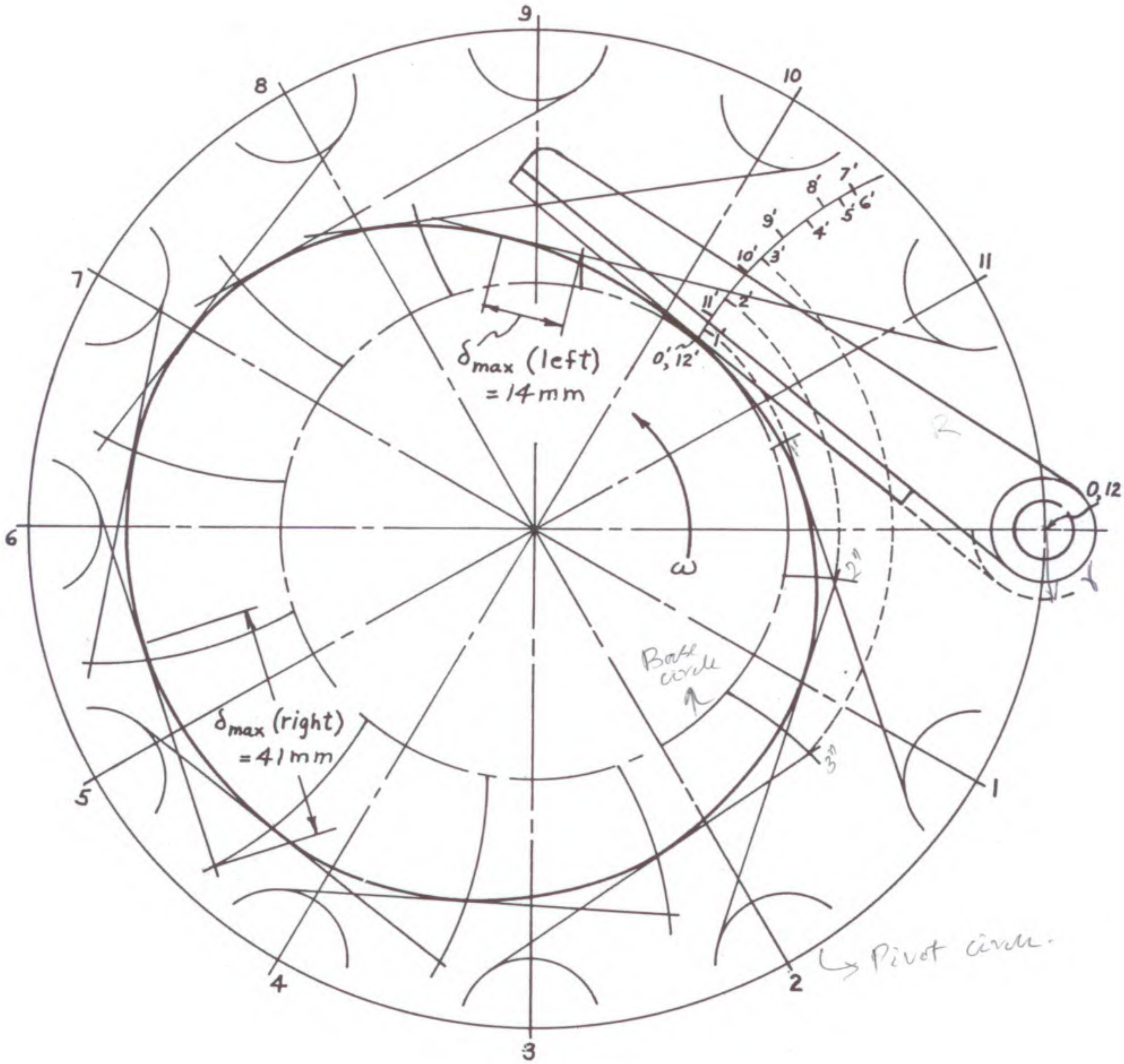


$\phi_{max} = 45^\circ$ approx.

10-11



10-12



9
8
10
7
11
6
5
4
3
2
1
0,12
0', 12'
1'
2'
3'
4'
5'
6'
7'
8'
9'
10'
11'
δ_{max} (left) = 14 mm
δ_{max} (right) = 41 mm
ω
Base circle
Pivot circle.

10-13

$$\beta = 95^\circ = 95\left(\frac{\pi}{180}\right) = 1.6581\ \text{rad}$$

$$s = h\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin 2\pi\frac{\theta}{\beta}\right)$$

$$\frac{ds}{d\theta} = \frac{h}{\beta}\left(1-\cos\frac{2\pi\theta}{\beta}\right) = \frac{25}{1.6581}\left(1-\cos\frac{2\pi\theta}{\beta}\right)$$

$$\tan\phi = \frac{ds}{r\,d\theta}$$

θ	θ/β	$2\pi\frac{\theta}{\beta}$	$\sin 2\pi\frac{\theta}{\beta}$
0	0	0	0
19°	0.2	72°	0.9510
38°	0.4	144°	0.5878
57°	0.6	216°	−0.5878
76°	0.8	288°	−0.9511
95°	1	360°	0

θ	$\cos 2\pi\frac{\theta}{\beta}$	$\sin\frac{2\pi\theta/\beta}{2\pi}$	s
0	1	0	0
19°	0.3091	0.1514	1.215
38°	−0.8090	0.0936	7.660
57°	−0.8090	−0.0936	17.340
76°	−0.3090	−0.1514	23.785
95°	1	0	25

θ	$r = R_b + s$	$ds/d\theta$	$\tan\phi$	ϕ
0	100	0	0	0
19°	101.215	10.417	0.1029	5.88°
38°	107.660	27.275	0.2533	14.21°
57°	117.340	27.275	0.2324	13.08°
76°	123.785	10.419	0.0842	4.81°
95°	125	0	0	0

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(600)}{60} = 62.8319\ \text{rad/s}$$

$$A_{max} = \frac{2\pi h\omega^2}{\beta^2} = \frac{2\pi(0.025)62.8319^2}{1.6581^2}$$

$$= \underline{\underline{225.6\ \text{m/s}^2}}$$

10-14

$$\omega = 600\ \text{r/min} = 600\left(\frac{2\pi}{60}\right) = 6.832\ \text{rad/s}$$

$$\beta = 95^\circ = 95\left(\frac{\pi}{180}\right) = 1.6581\ \text{rad}$$

$$h = 25\ \text{mm}$$

$$s = \frac{h}{2}\left(1-\cos\frac{\pi\theta}{\beta}\right) \longrightarrow (10.8)$$

$$\frac{ds}{d\theta} = \frac{h\pi}{2\beta}\left(\sin\frac{\pi\theta}{\beta}\right) \quad \text{—}(10.14)$$

$$r = R_b + s\ ; \quad \tan\phi = \frac{ds}{r\,d\theta}$$

—(10.15) $R_b \rightarrow$ Radius of base circle

θ	$\frac{\pi\theta}{\beta}$, deg	$\cos\frac{\pi\theta}{\beta}$	$\sin\frac{\pi\theta}{\beta}$	s
0	0	1	0	0
19°	36	0.8090	0.5878	2.388
38°	72	0.3090	0.9511	8.638
57°	108	−0.3090	0.9511	16.363
76°	144	−0.8090	0.5878	22.612
95°	180	−1	0	25

θ	$r = R_b + s$	$\frac{ds}{d\theta}$	$\tan\phi$	ϕ
0	100	0	0	0
19°	102.388	13.921	0.1360	7.74°
38°	108.638	22.526	0.2074	11.72°
57°	116.363	22.526	0.1936	10.96°
76°	122.612	13.921	0.1135	6.48°
95°	125	0	0	0

$$A_{max} = \frac{\pi^2 h\omega^2}{2\beta^2} = \frac{\pi^2(0.025)(62.832)^2}{2(1.6581)^2}$$

$$= \underline{\underline{177\ \text{m/s}^2}}$$

$$A = \frac{\pi^2 h\omega^2}{2\beta^2}\cos\frac{\pi\theta}{\beta} \qquad (10.10)$$

10-15

θ	r^2	$\sin\phi$	$\cos\phi$	$2r(R_g - R_r) \times \cos\phi$
0	10 000	0	1	9 400
19°	10 244	0.1024	0.9947	9 467
38°	11 591	0.2455	0.9694	9 810
57°	73769	0.2263	0.9741	10 744
76°	15 323	0.0839	0.9965	11 595
95°	15 625	0	1	11 750

θ	r_g^2	r_g	$\sin\eta$	η	ψ_g
0	21 609	147	0	0	0
19°	21 920	148.054	0.0325	1.862°	17.138°
38°	23 610	153.655	0.0751	4.307°	33.693°
57°	26722	163.469	0.0651	3.733°	53.267°
76°	29 127	170.666	0.0231	1.324	74.676°
95°	29 584	172	0	0	95°

10-16

$\beta = 95° = 1.6581 \text{ rad}$

$$s = h\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin 2\pi\frac{\theta}{\beta}\right)$$

$$q = \frac{ds}{d\theta} = \frac{h}{\beta}\left(1 - \cos\frac{2\pi\theta}{\beta}\right)$$

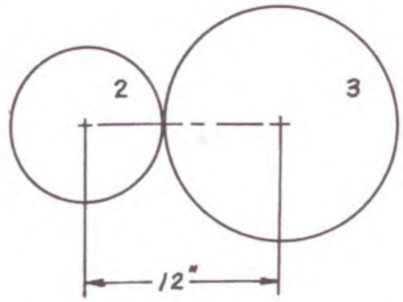
$$= \frac{25}{1.6581}\left(1 - \cos\frac{2\pi\theta}{\beta}\right)$$

θ	θ/β	$2\pi\frac{\theta}{\beta}$	$\sin\frac{2\pi\theta}{\beta}$	$\cos\frac{2\pi\theta}{\beta}$
0	0	0	0	1
19°	0.2	1.2566	0.9510	0.3091
38°	0.4	2.5133	0.5878	−0.8090
57°	0.6	3.7699	−0.5878	−0.8090
76°	0.8	5.0266	−0.9510	0.3091
95°	1	6.2832	0	1

θ	s	q	r_c	$\tan\eta$	η
0	0	0	100	0	0
19°	1.215	10.417	101.750	0.1029	5.875°
38°	7.660	27.275	111.061	0.2533	14.214°
57°	17.340	27.275	120.468	0.2324	13.083°
76°	23.785	10.417	124.223	0.0842	4.813°
95°	25	0	125	0	0

θ	ψ_c	r_g	$\cos\delta$	δ	ψ_g
0	0	162	1	0	0
19°	24.875°	163.547	0.9992	2.326°	22.549°
38°	52.214°	171.834	0.9961	5.092°	47.122°
57°	70.083°	181.402	0.9970	4.438°	65.645°
76°	80.813°	186.077	0.9996	1.603°	79.210°
95°	95°	187	1	0	95°

11-1



$$\frac{\omega_2}{\omega_3} = 1.5$$

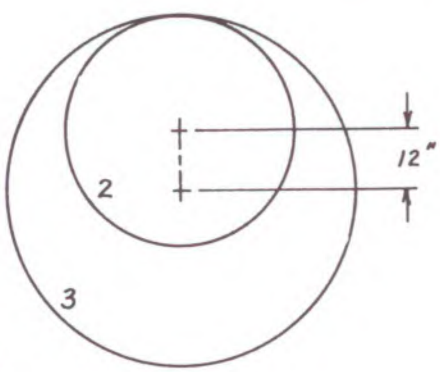
$$R_2 = \frac{C}{\frac{\omega_2}{\omega_3} + 1} = \frac{12}{1.5 + 1} = 4.80''$$

$$D_2 = 2(4.80) = \underline{9.60''}$$

$$R_3 = C - R_2 = 12 - 4.80 = 7.20''$$

$$D_3 = 2(7.20) = \underline{14.40''}$$

11-2



$$\frac{\omega_2}{\omega_3} = 1.5$$

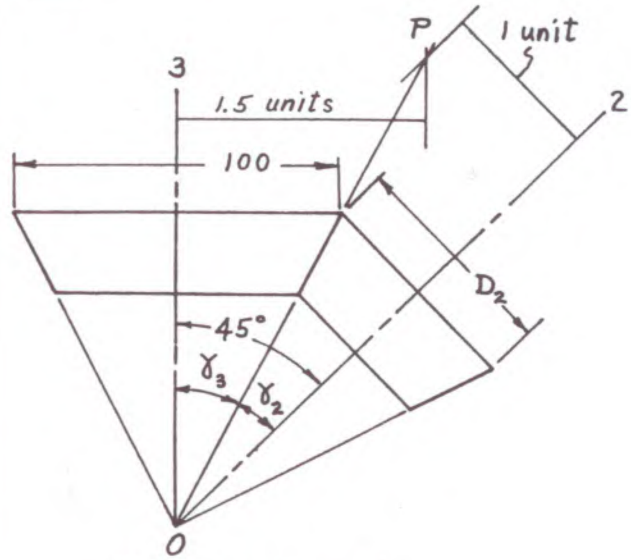
$$R_2 = \frac{C}{\frac{\omega_2}{\omega_3} - 1} = \frac{12}{1.5 - 1} = 24''$$

$$D_2 = 2(24) = \underline{48''}$$

$$R_3 = C + R_2 = 12 + 24 = 36''$$

$$D_3 = 2(36) = \underline{72''}$$

11-3

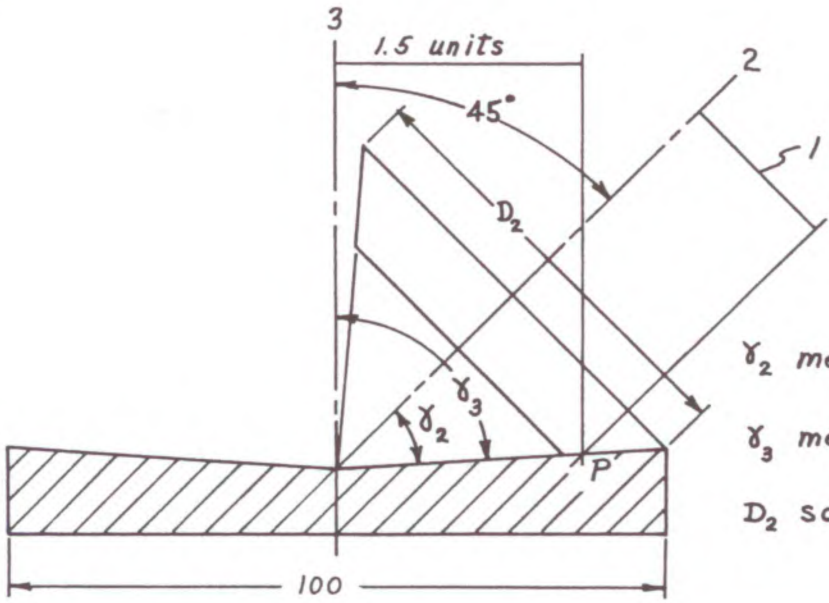


γ_2 measures $\underline{18°}$

γ_3 measures $\underline{27°}$

D_2 scales $\underline{67 mm}$

11-4



γ_2 measures $\underline{41^\circ}$

γ_3 measures $\underline{86^\circ}$

D_2 scales $\underline{67\text{mm}}$

11-5

$$\tan \gamma_3 = \frac{\sin \Sigma}{\frac{\omega_3}{\omega_2} + \cos \Sigma}$$

$$= \frac{0.707}{\frac{1}{1.5} + 0.707} = 0.515$$

$$\gamma_3 = \tan^{-1} 0.515 = \underline{27.23^\circ}$$

$$\gamma_2 = 45 - 27.23 = \underline{17.77^\circ}$$

$$D_2 = D_3 \frac{\omega_3}{\omega_2} = 100\left(\frac{1}{1.5}\right) = \underline{\underline{66.7\text{mm}}}$$

11-6

$$\tan \gamma_3 = \frac{\sin \Sigma}{\frac{\omega_3}{\omega_2} - \cos \Sigma}$$

$$= \frac{0.707}{\frac{1}{1.5} - 0.707} = 17.68$$

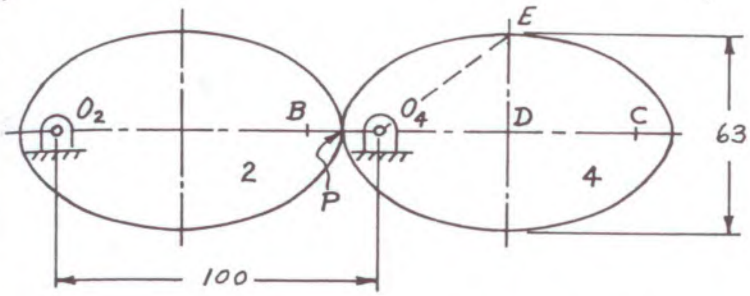
$$\gamma_3 = \tan^{-1} 17.68 = \underline{86.76^\circ}$$

$$\gamma_2 = 86.76 - 45 = \underline{41.76^\circ}$$

$$D_2 = D_3 \frac{\omega_3}{\omega_2} = 100\left(\frac{1}{1.5}\right) = \underline{\underline{66.7\text{mm}}}$$

11-7

a)



$$(O_4D)^2 = (O_4E)^2 - (DE)^2$$

$$= \left(\frac{100}{2}\right)^2 - (31.5)^2$$

$$= 1508$$

$$O_4D = \sqrt{1508} = \underline{\underline{38.8 \text{ mm}}}$$

b) $V.R. = \dfrac{\omega_2}{\omega_4}$; $V.R._{min.} = \dfrac{O_4P}{O_2P} = \dfrac{50-38.8}{100-(50-38.8)} = \dfrac{11.2}{88.8} = \underline{\underline{0.126}}$

$$V.R_{max.} = \frac{88.8}{11.2} = \underline{\underline{7.93}}$$

11-8

See figure in solution to Prob. 11-7.

Major axis $= O_2O_4 = \underline{254 \text{ mm}}$

$V.R. = \dfrac{\omega_2}{\omega_4}$; $V.R._{min.} = \dfrac{O_4P}{O_2P} = 0.143$ or $O_4P = 0.143(O_2P)$

But $O_4P + O_2P = 254$

$0.143(O_2P) + O_2P = 254$ $\quad 1.143(O_2P) = 254$

$$O_2P = \frac{254}{1.143} = 222 \text{ mm}$$

$$O_4P = 254 - O_2P = 254 - 222 = 32 \text{ mm}$$

$$O_4E = \frac{\text{major axis}}{2} = 127 \text{ mm}$$

$$O_4D = \frac{O_2P - O_4P}{2} = \frac{222-32}{2} = 95 \text{ mm}$$

$$DE = \sqrt{(O_4E)^2 - (O_4D)^2} = \sqrt{(127)^2 - (95)^2} = 84.3 \text{ mm}$$

Minor axis $= 2(DE) = 2(84.3) = \underline{\underline{168.6 \text{ mm}}}$

12-1

N_1, N_2
P, n

a) $D_1 = \dfrac{N}{P} = \dfrac{22}{8} = \underline{2.750 \text{ in}}$

$D_2 = \dfrac{38}{8} = \underline{4.750 \text{ in}}$

b) $C = \dfrac{D_1 + D_2}{2}$

$= \dfrac{2.750 + 4.750}{2} = \underline{3.750 \text{ in}}$

c) $p = \dfrac{\pi}{P} = \dfrac{3.141}{8} = \underline{0.393 \text{ in}}$

d) $V = \pi D n = \pi \dfrac{2.750}{12}(1800)$

$= \underline{1295 \text{ ft/min}}$

e) $\omega_2 = \dfrac{N_1}{N_2}\omega_1 = \dfrac{22}{38}(1800)$

$= \underline{1042 \text{ r/min}}$

12-2

$C = \dfrac{D_2 + D_3}{2} = 2.6 \qquad (1)$

$\dfrac{\omega_2}{\omega_3} = \dfrac{D_3}{D_2} = 1.6$

or

$D_3 = 1.6\, D_2 \qquad (2)$

Substitute (2) into (1).

$\dfrac{D_2 + 1.6\, D_2}{2} = 2.6$

$2.6\, D_2 = 2(2.6) = 5.2$

$D_2 = \dfrac{5.2}{2.6} = 2.000 \text{ in}$

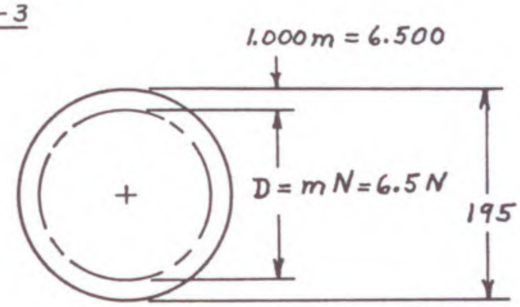
From (2)

$D_3 = 1.6\,(2.000) = 3.200 \text{ in}$

$N_2 = D_2 P = 2.000\,(10) = \underline{20 \text{ teeth}}$

$N_3 = D_3 P = 3.200\,(10) = \underline{32 \text{ teeth}}$

12-3

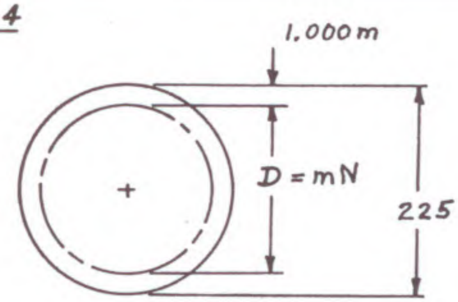


$6.5N + 2(6.500) = 195$

$6.5N + 13 = 195$

$N = \dfrac{182}{6.5} = \underline{28 \text{ teeth}}$

12-4



$mN + 2(1.000m) = 225$

$48m + 2m = 225$

$50m = 225 \quad \underline{m = 4.5 \text{ mm}}$

$p = \pi m = \pi(4.5) = \underline{14.137 \text{ mm}}$

12-5

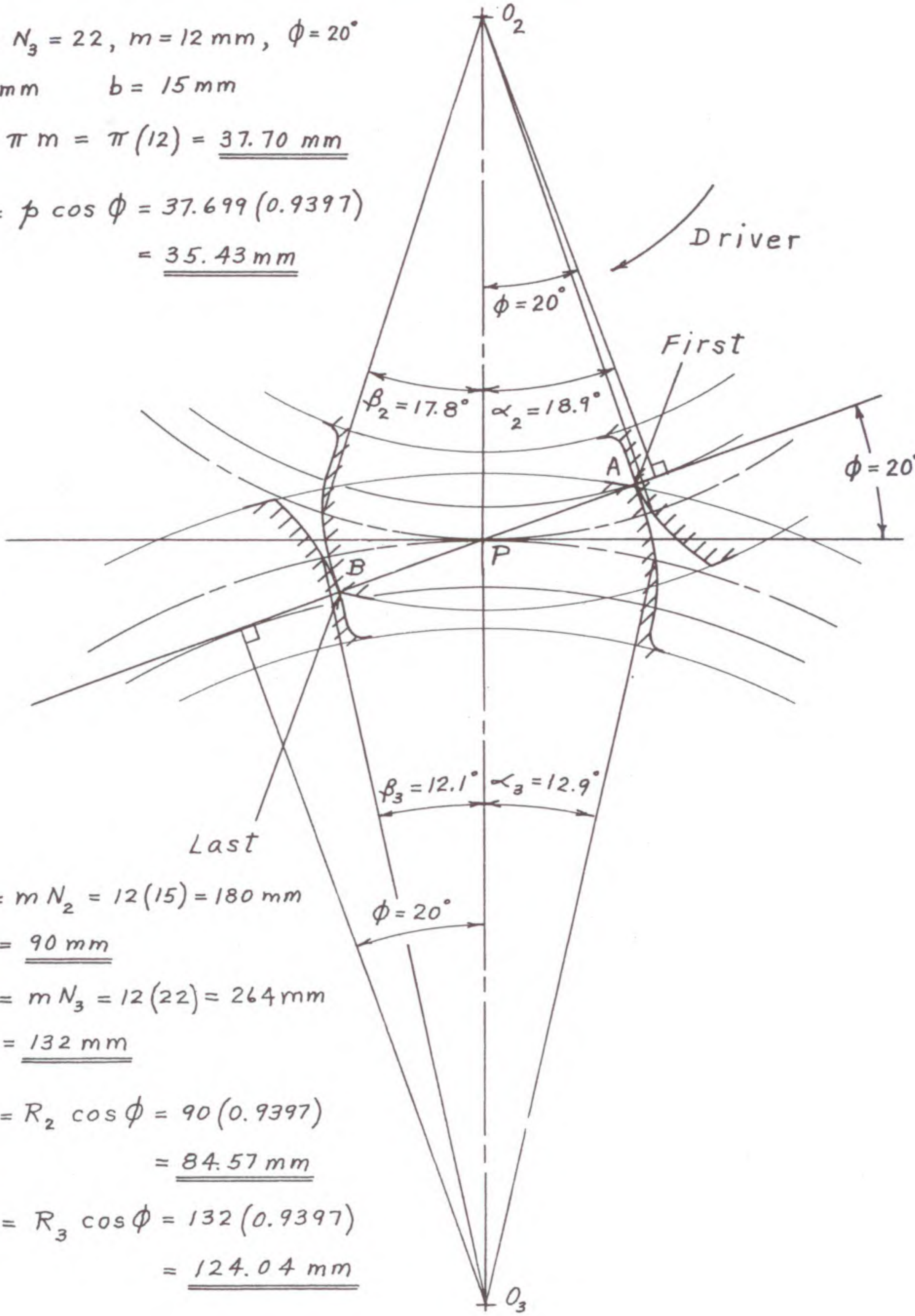
$N_2 = 15,\; N_3 = 22,\; m = 12\text{ mm},\; \phi = 20°$

$a = 12\text{ mm} \qquad b = 15\text{ mm}$

a) $p = \pi m = \pi(12) = \underline{\underline{37.70\text{ mm}}}$

b) $p_b = p\cos\phi = 37.699(0.9397)$

$= \underline{\underline{35.43\text{ mm}}}$



c) $D_2 = mN_2 = 12(15) = 180\text{ mm}$

$R_2 = \underline{\underline{90\text{ mm}}}$

$D_3 = mN_3 = 12(22) = 264\text{ mm}$

$R_3 = \underline{\underline{132\text{ mm}}}$

d) $r_2 = R_2\cos\phi = 90(0.9397)$

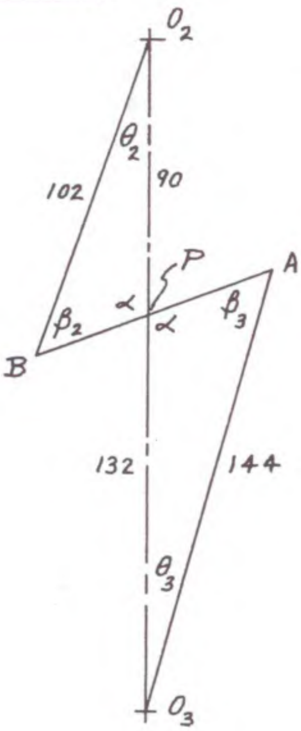
$= \underline{\underline{84.57\text{ mm}}}$

$r_3 = R_3\cos\phi = 132(0.9397)$

$= \underline{\underline{124.04\text{ mm}}}$

12-5 (contin.)

e)



$$\alpha = 90 + \phi = 110 \text{ deg}$$

$$\beta_3 = \sin^{-1} \frac{PO_3 \sin\alpha}{AO_3} = \sin^{-1} \frac{132(0.9397)}{144}$$

$$= \sin^{-1} 0.8614 = 59.5 \text{ deg}$$

$$\theta_3 = 180 - (\alpha + \beta_3) = 180 - (110 + 59.5) = 10.5 \text{ deg}$$

$$AP = \frac{AO_3 \sin\theta_3}{\sin\alpha} = \frac{144(0.1822)}{0.9397} = 27.92 \text{ mm}$$

$$\beta_2 = \sin^{-1} \frac{PO_2 \sin\alpha}{BO_2} = \sin^{-1} \frac{90(0.9397)}{102}$$

$$= \sin^{-1} 0.8291 = 56.0 \text{ deg}$$

$$\theta_2 = 180 - (\alpha + \beta_2) = 180 - (110 + 56) = 14 \text{ deg}$$

$$BP = \frac{BO_2 \sin\theta_2}{\sin\alpha} = \frac{102(0.2419)}{0.9397} = 26.26 \text{ mm}$$

Length of path of contact, $AB = AP + PB$

$$= 27.92 + 26.26 = \underline{\underline{54.18 \text{ mm}}}$$

f) $m_c = \dfrac{AB}{p_b} = \dfrac{54.19}{35.43} = \underline{\underline{1.53}}$

g) Angle of approach, pinion

$$\alpha_2 = \frac{AP}{r_2} = \frac{27.92}{84.57} = 0.3301 \text{ rad}$$

$$= \underline{\underline{18.9 \text{ deg}}}$$

Angle of approach, gear

$$\alpha_3 = \frac{AP}{r_3} = \frac{27.92}{124.0} = 0.2252 \text{ rad}$$

$$= \underline{\underline{12.9 \text{ deg}}}$$

Angle of recess, pinion

$$\beta_2 = \frac{PB}{r_2} = \frac{26.26}{84.57} = 0.3105 \text{ rad}$$

$$= \underline{\underline{17.8 \text{ deg}}}$$

Angle of recess, gear

$$\beta_3 = \frac{PB}{r_3} = \frac{26.26}{124.0} = 0.2118 \text{ rad}$$

$$= \underline{\underline{12.1 \text{ deg}}}$$

12-6

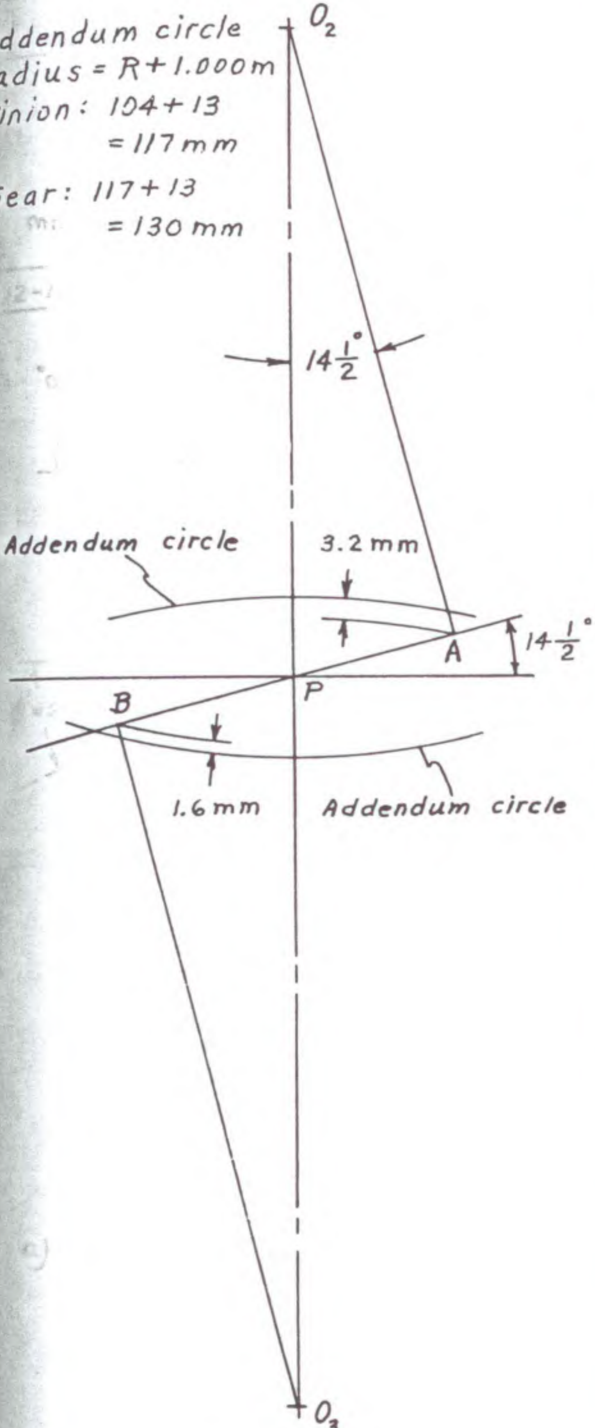
$D_2 = mN_2 = 13(16) = 208\text{ mm}$

$D_3 = mN_3 = 13(18) = 234\text{ mm}$

Addendum circle radius $= R + 1.000m$

Pinion: $104 + 13 = 117\text{ mm}$

Gear: $117 + 13 = 130\text{ mm}$



12-6 (CONT.)

$$p = \pi m = \pi(13) = 40.841\text{ mm}$$

$$p_b = p\cos\phi = 40.841(0.9681) = 39.538\text{ mm}$$

$$m_c = \frac{AB}{p_b} = \frac{56.5}{39.538} = \underline{\underline{1.42}}$$

12-7

$k = 1$

$$N = \frac{2k}{\sin^2\phi} = \frac{2}{(0.3827)^2} = \frac{2}{0.1464} = 13.66 \quad \text{Thus } \underline{\underline{14 \text{ teeth}}}$$

12-8

$k = 1$

$$N = \frac{2k}{\sin^2\phi} = \frac{2}{(0.4226)^2} = \frac{2}{0.1786} = 11.2 \quad \text{Thus } \underline{\underline{12 \text{ teeth}}}$$

12-9

Gear:

$$D = mN = 6.5(32) = 208\text{ mm}$$

$$PO_3 = \frac{D}{2} = 104\text{ mm}$$

$$AO_3 = PO_3 + a = 104 + 5.5 = 109.5\text{ mm}$$

$$\alpha = 110°$$

$$\beta_3 = \sin^{-1}\frac{PO_3 \sin\alpha}{AO_3} = \sin^{-1}\frac{104(0.9397)}{109.5} = \sin^{-1} 0.8925 = 63.2°$$

$$\theta_3 = 180 - (\alpha + \beta_3) = 180 - (110 + 63.2) = 6.8°$$

$$AP = \frac{AO_3 \sin\theta_3}{\sin\alpha}$$

12-9 (CONT.)

$$AP = \frac{109.5\,(0.1184)}{0.9397} = 13.80\text{ mm}$$

$$PO_2' = \frac{AP}{\sin\phi} = \frac{13.80}{0.3420} = 40.35\text{ mm}$$

Dia. of smallest pinion

$= 2\left(PO_2'\right) = 80.70$ mm

Number of teeth $= D/m$

$= 80.70/6.5 = 12.42$

Thus <u>13 teeth</u>

12-10

$D_2 = m N_2 = 13(15) = 195$ mm

$R_2 = 97.5$ mm

Addendum $= 1.000\,m = 13$ mm

Radius $O_2B = 97.5 + 13$

$= 110.5$ mm

$D_3 = m N_3 = 13(18) = 234$ mm

$R_3 = 117$ mm

Radius $O_3A = 117 + 13 = 13$ mm

a) $p = \pi m = \pi(13) = \underline{\underline{40.840\text{ mm}}}$

$p_b = p\cos\phi$

$= 40.840\cos 14\tfrac{1}{2}^\circ$

$= 40.840\,(0.9681)$

$= \underline{\underline{39.537\text{ mm}}}$

$$m_c = \frac{AB}{p_b} = \frac{66.2}{39.5} = \underline{\underline{1.68}}$$

b) $p = 40.840$ mm

$p_b = p\cos 20^\circ = 40.840\,(0.9397)$

$= 38.377$ mm

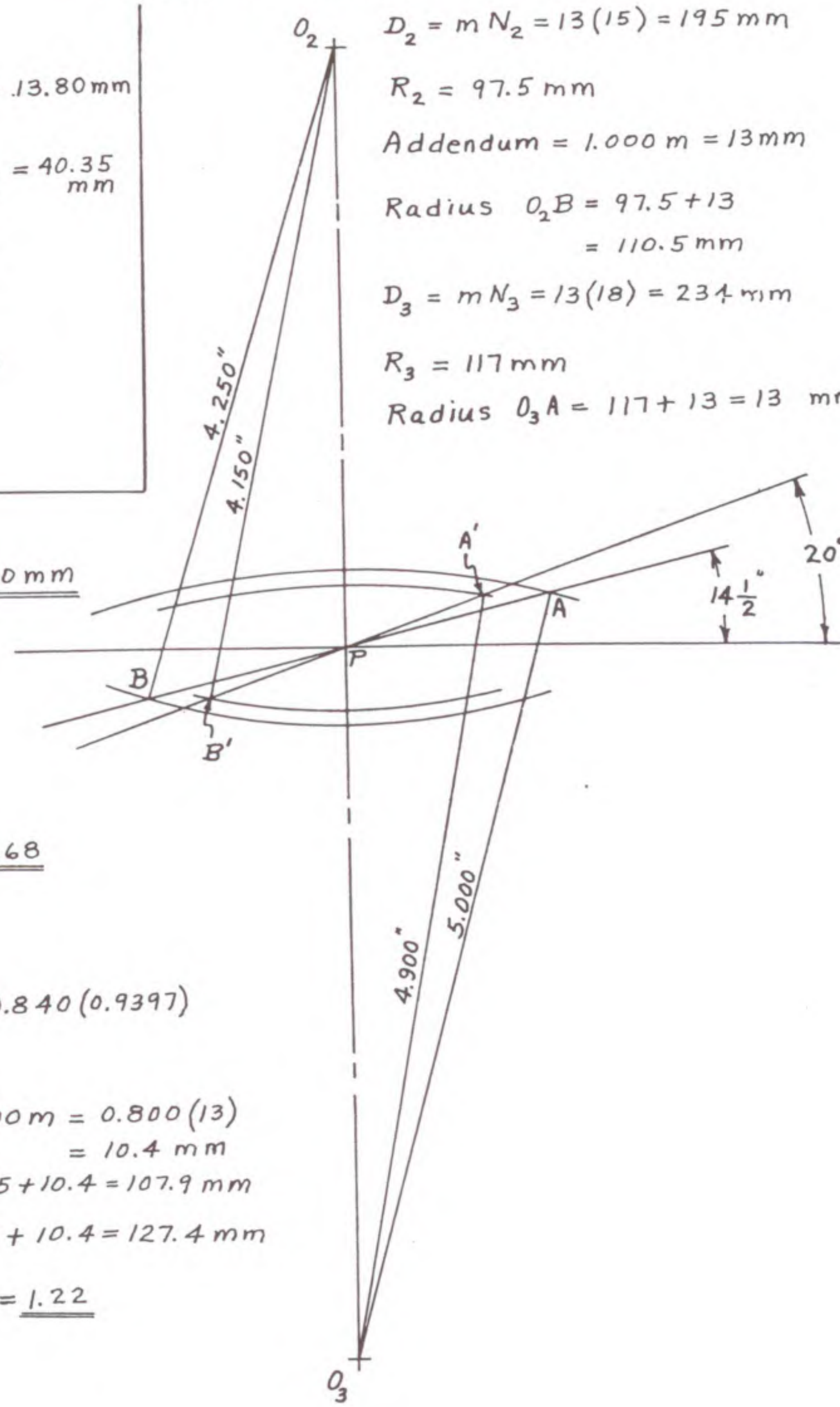
Addendum $= 0.800\,m = 0.800\,(13)$

$= 10.4$ mm

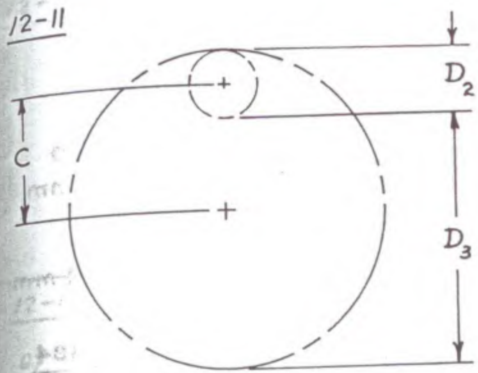
Radius $O_2B' = 97.5 + 10.4 = 107.9$ mm

Radius $O_3A' = 117 + 10.4 = 127.4$ mm

$$m_c = \frac{A'B'}{p_b} = \frac{47}{38.4} = \underline{\underline{1.22}}$$



12-11



a) $\dfrac{\omega_2}{\omega_3} = \dfrac{N_3}{N_2} = \dfrac{200}{40} = \underline{\underline{5}}$

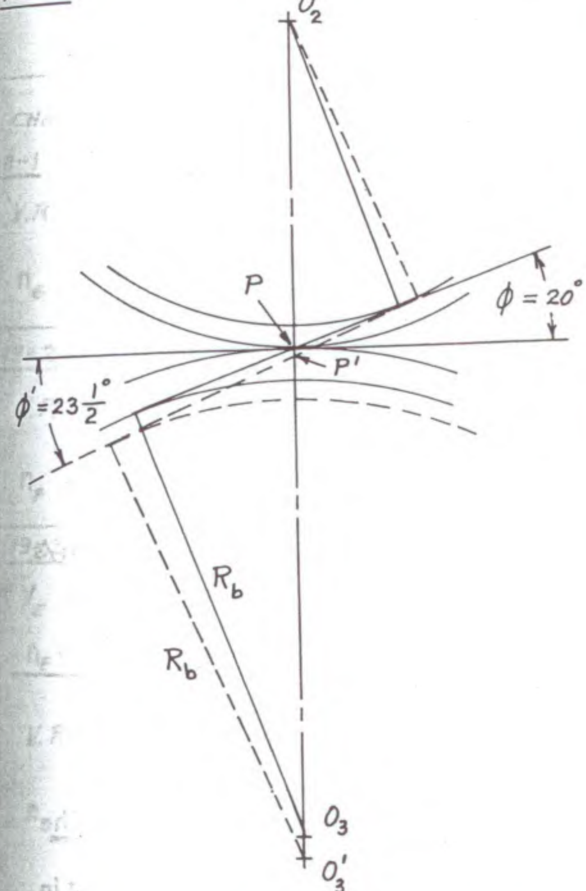
b) $D_2 = m N_2 = 2.5(40) = 100 \text{ mm}$

$D_3 = m N_3 = 2.5(200) = 500 \text{ mm}$

$C = \dfrac{D_3}{2} - \dfrac{D_2}{2}$

$= \dfrac{500-100}{2} = \underline{\underline{200 \text{ mm}}}$

12-12



a) $D_2 = m N_2 = 6.5(16) = 104 \text{ mm}$

$D_3 = m N_3 = 6.5(24) = 156 \text{ mm}$

$C = R_2 + R_3 = 52 + 78 = \underline{\underline{130 \text{ mm}}}$

12-12 (CONT.)

b) $O_2 O_3' = 130 + 3 = 133 \text{ mm}$

$O_3' P' = 133\left(\dfrac{78}{130}\right) = 79.8 \text{ mm}$

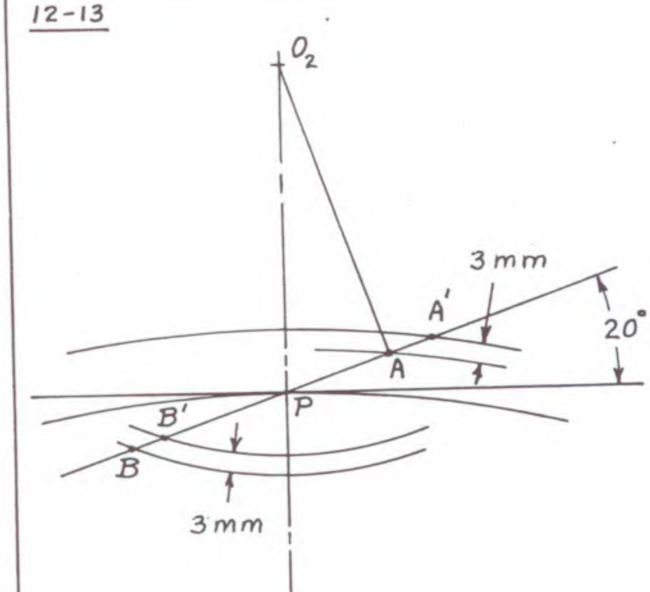
$R_b = R_3 \cos 20° = 78(0.9397)$

$= 73.297 \text{ mm}$

$\cos \phi' = \dfrac{R_b}{O_3' P'} = \dfrac{73.297}{79.8}$

$= 0.9185, \quad \underline{\phi' = 23.3°}$

12-13



12-13 (CONT.)

$D_2 = mN_2 = 10(10) = 100 \text{ mm}$

$D_3 = mN_3 = 10(35) = 350 \text{ mm}$

$a = 1.000m = 1.000(10) = 10 \text{ mm}$

$p = \pi m = \pi(10) = 31.416 \text{ mm}$

$p_b = p\cos 20^\circ$
$= 31.416\ (0.9397) = 29.522 \text{ mm}$

$A'B'$ = path with interference

AB = revised path

$m_c = \dfrac{AB}{p_b} = \dfrac{41.8}{31.416} = \underline{\underline{1.33}}$

12-14

$\psi = 23^\circ$

a) $m = \dfrac{m_n}{\cos\psi} = \dfrac{3}{0.9205} = \underline{\underline{3.259 \text{ mm}}}$

b) $D_2 = mN_2 = 3.259(30) = \underline{\underline{97.770 \text{ mm}}}$

$D_3 = mN_3 = 3.259(48) = \underline{\underline{156.432 \text{ mm}}}$

c) $C = \dfrac{D_2 + D_3}{2} = \dfrac{97.770 + 156.432}{2}$
$= \underline{\underline{127.101 \text{ mm}}}$

d) $p_n = \pi m_n = \pi(3) = \underline{\underline{9.425 \text{ mm}}}$

e) $p = \dfrac{p_n}{\cos\psi} = \dfrac{9.425}{0.9205} = \underline{\underline{10.239 \text{ mm}}}$

f) $F = \dfrac{1.15p}{\tan\psi} = \dfrac{1.15(10.239)}{0.4245} = \underline{\underline{27.74 \text{ mm}}}$

12-15

a) $\Sigma = \psi_2 + \psi_3$
$\psi_3 = \Sigma - \psi_2 = 45 - 20 = \underline{\underline{25^\circ}}$

b) For pinion and gear
$p_n = \pi m_n = \pi(2.5) = \underline{\underline{7.854 \text{ mm}}}$

c) $m_2 = \dfrac{m_n}{\cos\psi_2} = \dfrac{2.5}{0.9397} = \underline{\underline{2.660 \text{ mm}}}$

12-15 (CONTIN.)

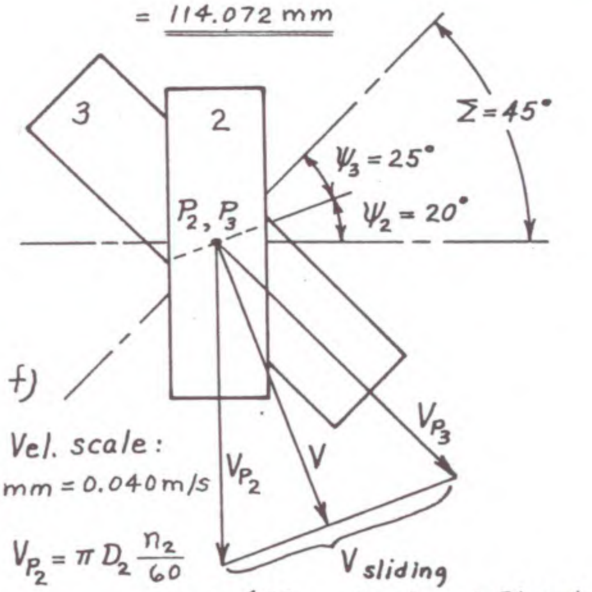
d) $m_3 = \dfrac{m_n}{\cos\psi_3}$
$= \dfrac{2.5}{0.9063} = 2.758 \text{ mm}$

e) $D_2 = m_2N_2 = 2.660(36) = 95.760 \text{ mm}$

$D_3 = m_3N_3 = 2.758(48) = 132.384 \text{ mm}$

$C = \dfrac{D_2 + D_3}{2} = \dfrac{95.760 + 132.384}{2}$
$= \underline{\underline{114.072 \text{ mm}}}$

f)



Vel. scale:
1 mm = 0.040 m/s

$V_{P_2} = \pi D_2 \dfrac{n_2}{60}$
$= \pi(0.0958)\dfrac{400}{60}$
$= \underline{\underline{2.01 \text{ m/s}}}$

$V_{sliding}$ scales 1.56 m/s

12-16

a) $\dfrac{\omega_w}{\omega_g} = \dfrac{N_g}{N_w} = \dfrac{15}{1}$

$N_g = 15N_w = 15(3) = \underline{45 \text{ teeth}}$

b) $D_g = \dfrac{pN_g}{\pi} = \dfrac{10(45)}{\pi} = \underline{\underline{5.73 \text{ in}}}$

c) $\psi_g = \lambda_w = \underline{\underline{20^\circ}}$

d) $\tan\lambda = \dfrac{N_w p}{\pi D_w}$

$D_w = \dfrac{N_w p}{\pi\tan\lambda} = \dfrac{3(10)}{\pi\tan 20^\circ}$

12-16 (CONT.)

$$D_w = \frac{1.2}{\pi(0.3640)} = \underline{\underline{1.049\text{ in}}}$$

e) $$C = \frac{D_w + D_g}{2} = \frac{1.049+5.73}{2}$$

$$= \underline{\underline{3.390\text{ in}}}$$

12-17

a) $$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{36}{18} = \underline{\underline{2}}$$

b) $$D_2 = \frac{N_2}{P} = \frac{18}{6} = \underline{\underline{3\text{ in}}}$$

$$D_3 = \frac{N_3}{P} = \frac{36}{6} = \underline{\underline{6\text{ in}}}$$

12-17 (CONT.)

c) $R_2 = 1.5\text{ in}$, $R_3 = 3\text{ in}$

$$\tan\gamma_3 = \frac{R_3}{R_2} = \frac{3}{1.5} = 2$$

$$\underline{\gamma_3 = 63.43^\circ}$$

d) $$\text{Cone distance} = \frac{R_3}{\sin\gamma_3}$$

$$= \frac{3}{0.8944} = \underline{\underline{3.36\text{ in}}}$$

e) $$R_{b_3} = \frac{R_3}{\cos\gamma_3} = \frac{3}{0.4473} = 6.70\text{ in}$$

f) $$N_{b_3} = N_3\frac{R_{b_3}}{R_3} = 36\left(\frac{6.70}{3}\right)$$

$$= \underline{\underline{80.5\text{ teeth}}}$$

CHAPTER 13. GEAR TRAINS, TRANSLATION SCREWS, MECHANICAL ADVANTAGE

13-1

$$V.R. = \frac{40}{26}\times\frac{46}{28}\times\frac{42}{24}\times\frac{48}{42} = 5.05$$

$$n_G = \frac{n_A}{V.R.} = \frac{1200}{5.05} = \underline{238\text{ r/min cw}}$$

13-2

$$V.R. = \frac{36}{18}\times\frac{40}{2}\times\frac{56}{32} = 70$$

$$n_F = \frac{n_A}{V.R.} = \frac{900}{70} = \underline{12.9\text{ r/min cw}}$$

13-3

$$V_E = \pi D_E n_E$$

$$n_E = \frac{V_E}{\pi D_E} = \frac{2\times 60}{\pi\left(\frac{30}{12}\right)} = 15.3$$

$$V.R. = \frac{n_A}{n_D} = \frac{90}{3}\times\frac{N_D}{40} = 0.75\,N_D$$

$$n_D = \frac{n_A}{V.R.} = \frac{n_A}{0.75\,N_D}$$

$$15.3 = \frac{600}{0.75\,N_D}$$

$$N_D = \frac{600}{0.75\times 15.3} = 52.2$$

<u>Use 52 teeth</u>

<u>A turns ccw</u>

13-4

$$V.R = \frac{N_B}{N_A}\times\frac{N_D}{N_C} = 11.4 \qquad (1)$$

$$D = mN\,,\quad C = \frac{D_A + D_B}{2}$$

$$C = \frac{m(N_A+N_B)}{2}$$

$$150 = \frac{2.5(N_A+N_B)}{2}\,,\quad N_A+N_B = 120 \qquad (2)$$

$$150 = \frac{2(N_C+N_D)}{2}\,,\quad N_C+N_D = 150 \qquad (3)$$

Four unknowns, N_A, N_B, N_C, N_D, but only three equations (1), (2), and (3). Must solve by trial.

Trying $N_A = 24$, from Eq. (2)

$$N_B = 120 - 24 = 96$$

From Eq. (1)

$$\frac{96}{24}\frac{N_D}{N_C} = 11.4\,,\quad N_C = \frac{96\,N_D}{24(11.4)}$$

$$= 0.351\,N_D$$

From Eq. (3)

$$0.351\,N_D + N_D = 150\,,\quad N_D = 111.03$$

$$N_C = 0.351\,N_D = 0.351(111.03) = 38.97$$

Use $\underline{N_A = 24}$, $\underline{N_B = 96}$

$\underline{N_C = 39}$, $\underline{N_D = 111}$

$$V.R. = \frac{N_B}{N_A}\times\frac{N_D}{N_C} = \frac{96}{24}\times\frac{111}{39} = 11.385$$

$11.385 \cong 11.4$ satisfactory

13-5

a) Let V = vel. of point on wheel relative to axle.

$$V = \frac{60 \times 5280}{60} = 5280 \text{ ft/min}$$

$$\text{Wheel rpm} = \frac{V}{\pi D} = \frac{5280}{\pi\left(\frac{27}{12}\right)} = 748 \text{ r/min}$$

$$\text{Engine speed} = 3.5(748) = \underline{\underline{2620 \text{ r/min}}}$$

b)

Member	Arm	A	B	C
Train locked and arm given one positive turn	+1	+1	+1	+1
Arm fixed, A given one negative turn	0	−1	$+\frac{23}{16}$	$+\frac{23}{16} \times \frac{16}{55}$
Resultant turns	+1	0	+2.435	+1.418
	Driver			Driven

From results in table, $\dfrac{n_{arm}}{n_c} = \dfrac{1}{1.418} = 0.705$

Engine speed $= 0.705\,(2620) = \underline{\underline{1845 \text{ r/min}}}$

13-6

Member	Arm	A	B	C	D
Train locked and arm given one positive turn	+1	+1	+1	+1	+1
Arm fixed, A given one negative turn	0	−1	$+\frac{50}{51}$	$+\frac{50}{51}$	$-\left(\frac{50}{51} \times \frac{50}{51}\right)$
Resultant turns	+1	0	+1.980	+1.980	+0.0388

From results in table, $\dfrac{n_{arm}}{n_D} = \dfrac{1}{0.0388} = 25.8$

$\therefore$ speed reduction = $\underline{\underline{25.8 \text{ to } 1}}$

$\underline{\underline{\text{Output shaft rotates cw}}}$

13-7

Member	Arm	A	B	C	D	E
Train locked and arm given one positive turn	+1	+1	+1	+1	+1	+1
Arm fixed, D given one negative turn	0	$+\left(\frac{105}{30}\times\frac{25}{50}\right)$	$-\frac{105}{30}$	$-\frac{105}{30}$	−1	$+\left(\frac{105}{30}\times\frac{30}{45}\right)$
Resultant turns	+1	+2.75	−2.5	−2.5	0	+3.333

From results in table, $\frac{n_E}{n_A} = \frac{3.333}{2.75} = 1.21$

$n_E = 1.21\, n_A = 1.21(-150) = \underline{\underline{-181.5 \text{ r/min}}}$

Minus sign indicates cw

13-8

Let gear A be input 1, and arm be input 2. F is output.

$$n_o = n_1\left(\frac{n_o}{n_1}\right)_{\substack{\text{input 2}\\ \text{held fixed}}} + n_2\left(\frac{n_o}{n_2}\right)_{\substack{\text{input 1}\\ \text{held fixed}}} \qquad (1)$$

$$\left(\frac{n_o}{n_1}\right)_{\substack{\text{input 2}\\ \text{held fixed}}} = \frac{28}{32}\times\frac{58}{48}\times\frac{50}{56} = -0.944$$

Member	Arm	C	D	E	F
Train locked and arm given one positive turn	+1	+1	+1	+1	+1
Arm fixed, C given one negative turn	0	−1	$+\frac{58}{48}$	$+\frac{58}{48}$	$-\left(\frac{58}{48}\times\frac{50}{56}\right)$
Resultant turns	+1	0	+2.208	+2.208	−0.0789

From results in table, $\left(\frac{n_o}{n_2}\right)_{\substack{\text{input 1}\\ \text{held fixed}}} = \frac{n_F}{n_{arm}} = \frac{-0.0789}{+1} = -0.0789$

From Eq. (1) above, $n_o = -75(-0.944) + (-150)(-0.0789)$

$= +70.8 + 11.8 = \underline{+82.6 \text{ r/min}}$

Plus sign indicates ccw

13-9

a)

Member	Arm	A	B	C	D
Train locked and arm given one positive turn	+1	+1	---	---	+1
Arm fixed, A given one negative turn	0	-1	---	---	$-\left(\frac{301}{40} \times \frac{40}{300}\right)$
Resultant turns	+1	0	---	---	$-\frac{1}{300}$

From results in table,

$$\frac{n_{arm}}{n_D} = \frac{+1}{-\frac{1}{300}} = -300 \; ; \quad \therefore \text{ speed reduction is } \underline{\underline{300 \text{ to } 1}}$$

Minus sign indicates Arm and D rotate in opposite directions. $\therefore$ D rotates cw

b) $$\frac{n_{arm}}{n_D} = \frac{+1}{+\frac{1}{301}} = 301 \; ; \quad \therefore \text{ speed reduction is } \underline{\underline{301 \text{ to } 1}}$$

Plus sign indicates Arm and D rotate in same direction. $\therefore$ D rotates ccw

---

13-10

Let D be input 1 and F input 2.

$n_1 = n_D = +\left(\frac{40}{20} \times \frac{20}{30}\right) = +\frac{4}{3}$ turns $\qquad n_2 = n_F = -\frac{70}{40} = -\frac{7}{4}$ turns

$$n_o = \underbrace{n_1 \left(\frac{n_o}{n_1}\right)_{\substack{\text{input 2} \\ \text{held fixed}}}}_{\text{I}} + \underbrace{n_2 \left(\frac{n_o}{n_2}\right)_{\substack{\text{input 1} \\ \text{held fixed}}}}_{\text{II}} \qquad (1)$$

To determine I (F held fixed):

Member	Arm	D	E	F
Train locked and arm given one positive turn	+1	+1	---	+1
Arm fixed, F given one negative turn	0	$+\frac{50}{10}$	---	-1
Resultant turns	+1	+6	---	0

From results in table $\quad \frac{n_o}{n_1} = \frac{n_{arm}}{n_D} = \frac{1}{6}$

13-10 (CONT.)

To determine II (D held fixed):

Member	Arm	D	E	F
Train locked and arm given one positive turn	+1	+1	---	+1
Arm fixed, D given one negative turn	0	-1	---	$+\frac{10}{50}$
Resultant turns	+1	0	---	$+\frac{6}{5}$

From results in table $\frac{n_o}{n_2} = \frac{n_{arm}}{n_F} = \frac{+1}{+\frac{6}{5}} = \frac{5}{6}$

From Eq. (1) above

$$n_o = +\frac{4}{3}\left(\frac{1}{6}\right) - \frac{7}{4}\left(\frac{5}{6}\right) = +\frac{4}{18} - \frac{35}{24}$$

$$= +0.222 - 1.458 = \underline{-1.236}$$ Minus sign indicates cw.

13-11

If W moves up 1 ft, this produces 1 ft of slack in b and 1 ft in c. To remove this slack, F must move down 2 ft.

$$M.A. = \frac{S_i}{S_o} = \frac{2}{1} = \underline{2}$$

13-12

For 1 turn of upper pulleys F moves amount $S_i = \pi D_2$.
Amount of chain coming off small pulley at d is πD_1.
Amount of chain coming onto large pulley at a is πD_2.
Shortening of loop defa $= \pi D_2 - \pi D_1 = \pi (D_2 - D_1)$.

Amount W moves up is

$$S_o = \frac{\pi(D_2 - D_1)}{2}$$

$$M.A. = \frac{S_i}{S_o} = \frac{\pi D_2}{\frac{\pi}{2}(D_2 - D_1)}$$

$$= \underline{\frac{2D_2}{D_2 - D_1}}$$

13-13

$$M.A. = \frac{S_i}{S_o} = \frac{2\pi(200)n_A}{\pi(150)\,n_{arm}} = 24 \quad \text{or} \quad \frac{n_{arm}}{n_A} = \frac{2(200)}{150(24)} = \frac{1}{9}$$

Member	Arm	A	B	C
Train locked and arm given one positive turn	+1	+1	+1	+1
Arm fixed, C given one negative turn	0	$+\left(\frac{96}{N_B} \times \frac{N_B}{N_A}\right)$	$-\frac{96}{N_B}$	−1
Resultant turns	+1	$1 + \frac{96}{N_A}$	$1 - \frac{96}{N_B}$	0

From results in table, $\dfrac{N_{arm}}{n_A} = \dfrac{1}{1 + \frac{96}{N_A}}$

Then $\quad 1 + \dfrac{96}{N_A} = 9, \quad \dfrac{96}{N_A} = 8, \quad \underline{N_A = 12 \text{ teeth}}$

To satisfy center distance

$$R_A + D_B = R_C \qquad \text{or} \qquad D_A + 2D_B = D_C$$

Then

$$N_A m + 2N_B m = N_C m$$

$$N_A + 2N_B = 96$$

$$12 + 2N_B = 96, \qquad N_B = \frac{96-12}{2} = \frac{84}{2} = \underline{\underline{42 \text{ teeth}}}$$

13-14

$$S_i = 2\pi R = 2\pi(1.25) = 7.86 \text{ in.}$$

a) $S_o = \dfrac{1}{12} - \dfrac{1}{13} = \dfrac{1}{156}$

$$M.A. = E\,\frac{S_i}{S_o} = (1)\,\frac{7.86}{1/156} = \underline{1225}$$

b) $S_o = \dfrac{1}{12} + \dfrac{1}{13} = \dfrac{25}{156}$

$$M.A. = E\,\frac{S_i}{S_o} = (1)\,\frac{7.86}{25/156} = \underline{\underline{49}}$$

13-15

From the vel. diagram in Prob. 6-7

$V_{P_2} = 1.53$ m/s, V_{P_4} scales 0.463 m/s

$$M.A. = E \frac{V_i}{V_o}$$

$$= E \frac{V_{P_2}}{V_{P_4}} = (1) \frac{1.53}{0.463} = \underline{\underline{3.31}}$$

13-16

From the vel. diagram in Prob. 6-10

$V_B = 4.58$ m/s, V_E scales 9.45 m/s

$$M.A. = E \frac{V_i}{V_o}$$

$$= E \frac{V_B}{V_E} = (1) \frac{4.58}{9.45} = \underline{\underline{0.485}}$$

14-1

$$\theta_1 + \theta_2 = 360° \qquad (1)$$

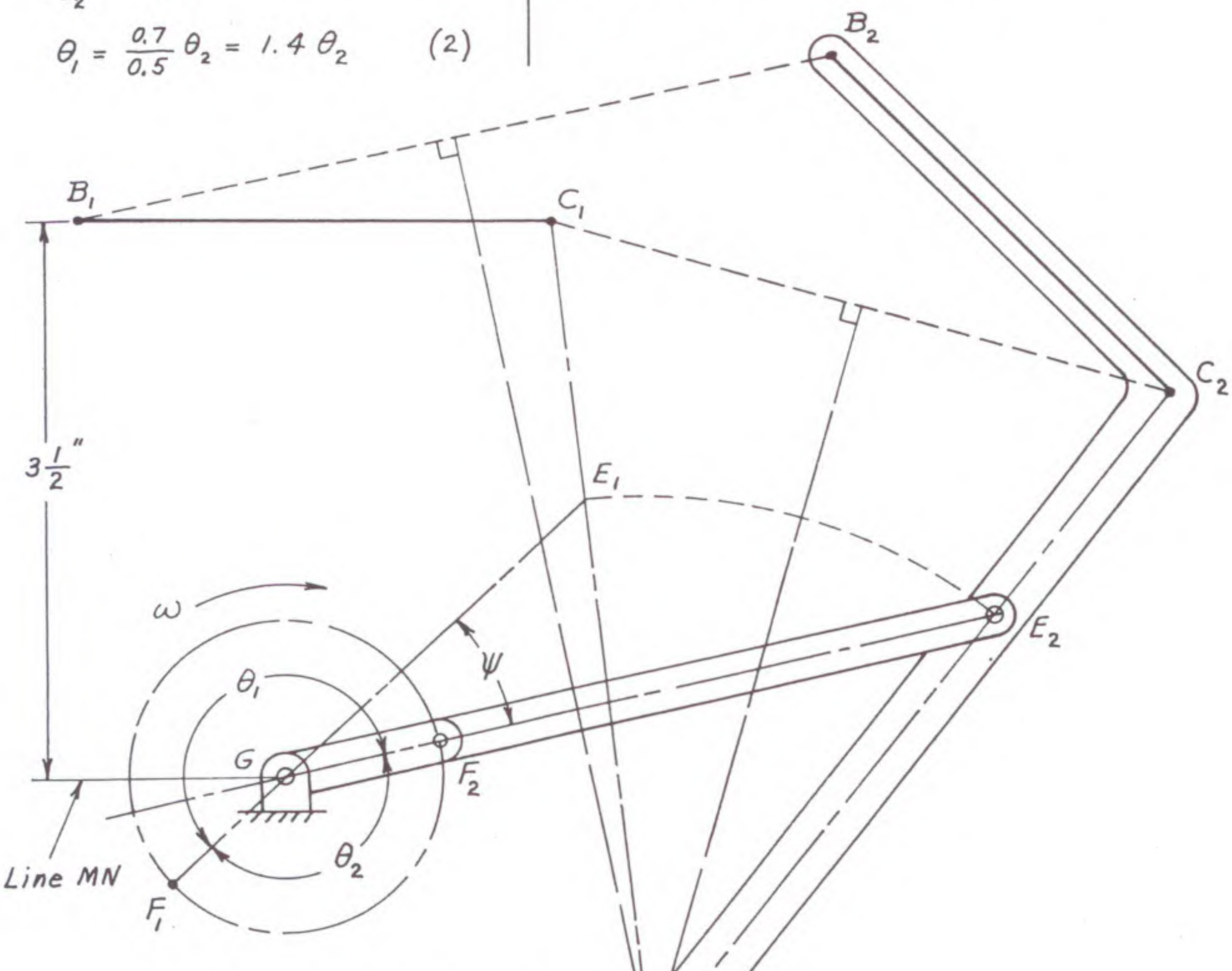
$$\frac{\theta_1}{\theta_2} = \frac{0.7}{0.5}$$

$$\theta_1 = \frac{0.7}{0.5}\theta_2 = 1.4\,\theta_2 \qquad (2)$$

Substitute Eq. (2) into (1)

$1.4\,\theta_2 + \theta_2 = 360°$, $2.4\,\theta_2 = 360°$, $\theta_2 = 150°$

$\theta_1 = 360° - \theta_2 = 360° - 150° = 210°$



$$\psi = \frac{\theta_1 - \theta_2}{2} = \frac{210 - 150}{2} = 30°$$

Move 30° triangle about until edges pass through E_1 and E_2 and vertex lies on line MN. G lies at vertex.

$0.7 + 0.5 = 1.2\,s$

$\frac{60}{1.2} = \underline{50\ r/min}$

14-2

Position	x	y	Degrees rotation from start: Driver ϕ, clockwise	Driven ψ, clockwise
0	−1	0.368	0	0
1	−0.5	0.607	25	10.2
2	0	1.000	50	26.9
3	0.5	1.649	75	54.5
4	1	2.718	100	100

$$\phi = \phi_s + \frac{\phi_f - \phi_s}{x_f - x_s}(x - x_s)$$

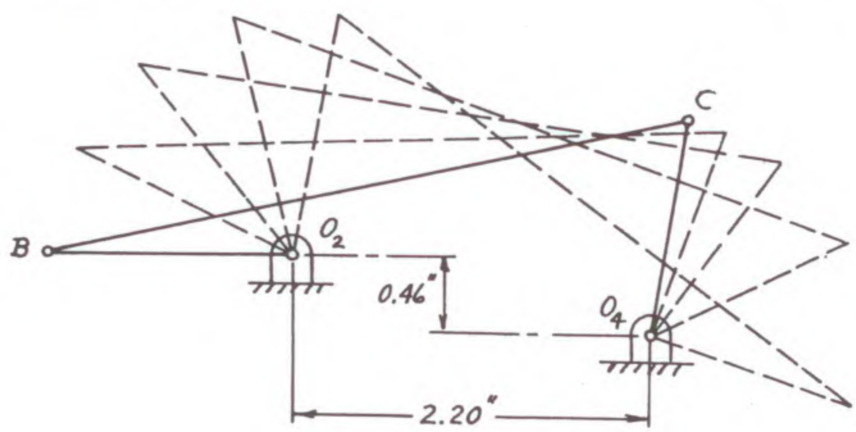
$$= 0 + \frac{100}{1-(-1)}\left[x-(-1)\right]$$

$$= 50(x+1)$$

$$\psi = \psi_s + \frac{\psi_f - \psi_s}{y_f - y_s}(y - y_s)$$

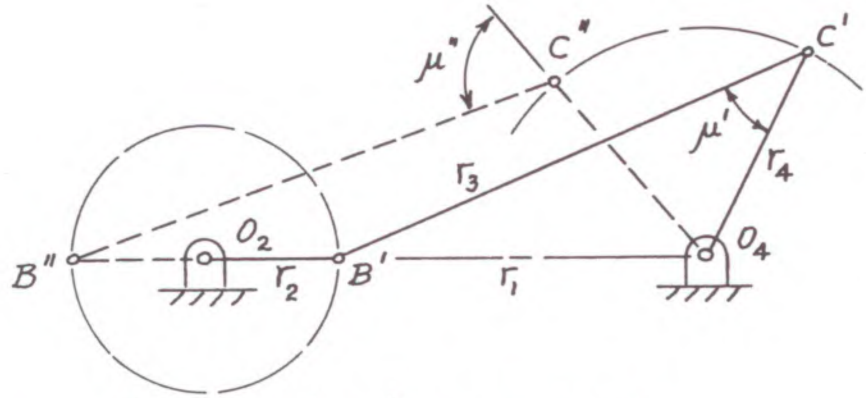
$$= 0 + \frac{100}{2.350}(y - 0.368)$$

$$= 42.55(y - 0.368)$$



$O_2B = 1.50"$
$BC = 4.00"$
$O_4C = 1.34"$

14-3



$$\cos\beta = \frac{r_1^2 + r_2^2 - r_3^2 - r_4^2 - 2 r_1 r_2 \cos\theta_2}{-2 r_3 r_4}$$

When crank is at B'

$$\cos\beta = \frac{130^2 + 34^2 - 133^2 - 60^2 - 2\,(130\times 34)\cos 0^\circ}{-2\times 133\times 60}$$

$$= \frac{16\,900 + 1\,156 - 17\,689 - 3\,600 - 8\,840}{-15\,960} = \frac{-12\,073}{-15\,960} = 0.7565$$

$$\beta = 40.85^\circ, \quad \underline{\underline{\mu' = 40.85^\circ}}$$

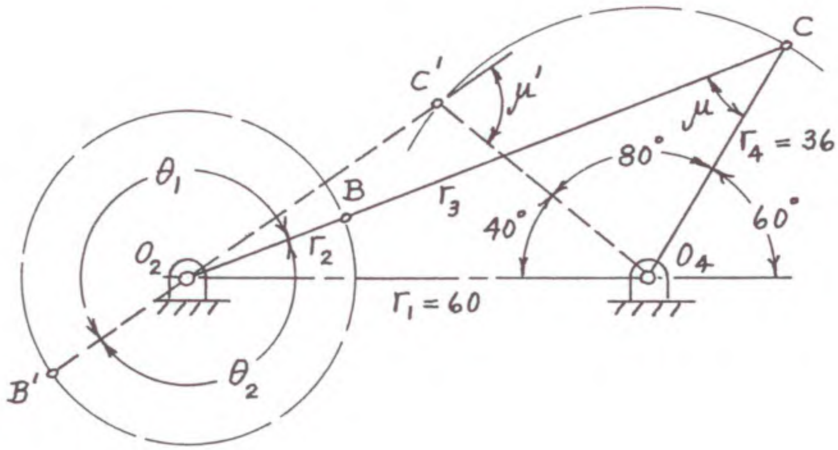
When crank is at B''

$$\cos\beta = \frac{130^2 + 34^2 - 133^2 - 60^2 - 2\,(130\times 34)\cos 180^\circ}{-2\times 133\times 60}$$

$$= \frac{16900 + 1156 - 17\,689 - 3\,600 + 8\,840}{-15\,960} = \frac{5607}{-15\,960} = -0.3513$$

$$\beta = 110.57^\circ, \quad \mu'' = 180 - 110.57 = \underline{\underline{69.43^\circ}}$$

14-4



For $\Delta\, O_2 O_4 C$

$$(r_2 + r_3)^2 = r_1^2 + r_4^2 - 2 r_1 r_4 \cos 120^\circ$$

$$= 60^2 + 36^2 - 2(60 \times 36)(-0.5)$$

$$= 3600 + 1296 + 2160 = 7056$$

$$r_2 + r_3 = \sqrt{7056} = 84 \qquad (1)$$

For $\Delta\, O_2 O_4 C'$

$$(r_3 - r_2)^2 = r_1^2 + r_4^2 - 2 r_1 r_4 \cos 40^\circ$$

$$= 60^2 + 36^2 - 2(60 \times 36)\,0.7660$$

$$= 3600 + 1296 - 3309 = 1587$$

$$r_3 - r_2 = \sqrt{1587} = 39.837 \qquad (2)$$

Adding Eqs. (1) and (2)

$$2 r_3 = 123.837, \quad \underline{\underline{r_3 = 61.919 \text{ mm}}}$$

From Eq. (1), $r_2 = 84 - 61.919 = \underline{\underline{22.081 \text{ mm}}}$

$\underline{\underline{\mu \text{ measures } 38^\circ}}$ $\qquad$ $\underline{\underline{\theta_2 \text{ measures } 166^\circ}}$ $\qquad$ $T.R. = \dfrac{\theta_1}{\theta_2} = \dfrac{194}{166}$

$\underline{\underline{\mu' \quad \text{"} \quad 76^\circ}}$ $\qquad$ $\underline{\underline{\theta_1 = 360 - 166 = 194^\circ}}$ $\qquad$ $= \underline{\underline{1.17}}$

14-5

x, deg	y	ϕ, deg	ψ, deg
0	0	300	250
60	0.866	360	328
90	1.000	390	340

$$\phi = \phi_s + \frac{\phi_f - \phi_s}{x_f - x_s}(x - x_s)$$

$$= 300 + \frac{90}{90-0}(x-0)$$

$$= 300 + x$$

$$\psi = \psi_s + \frac{\psi_f - \psi_s}{y_f - y_s}(y - y_s)$$

$$= 250 + \frac{90}{1-0}(y-0)$$

$$= 250 + 90y$$

$$R_1 \cos 300 - R_2 \cos 250 + R_3 = \cos 50$$

$$R_1 \cos 360 - R_2 \cos 328 + R_3 = \cos 32$$

$$R_1 \cos 390 - R_2 \cos 340 + R_3 = \cos 50$$

$$0.500 R_1 + 0.342 R_2 + R_3 = 0.643$$

$$R_1 - 0.848 R_2 + R_3 = 0.848$$

$$0.866 R_1 - 0.940 R_2 + R_3 = 0.643$$

$R_1 = 1.280$, $R_2 = 0.365$

$R_3 = -0.122$

Let $\underline{\underline{a = 25\text{ mm}}}$

$$d = \frac{a}{R_1} = \frac{25}{1.280} = \underline{\underline{19.53\text{ mm}}}$$

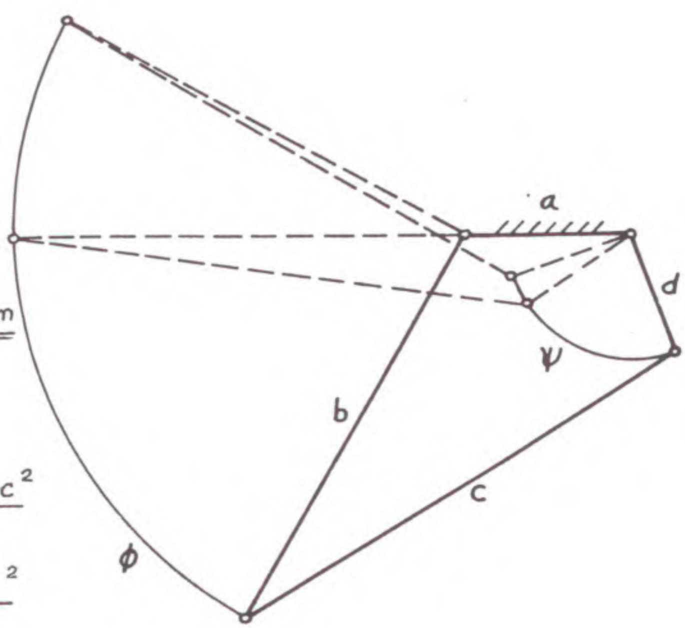
$$b = \frac{a}{R_2} = \frac{25}{0.365} = \underline{\underline{68.49\text{ mm}}}$$

$$R_3 = \frac{a^2 + b^2 + d^2 - c^2}{2bd}$$

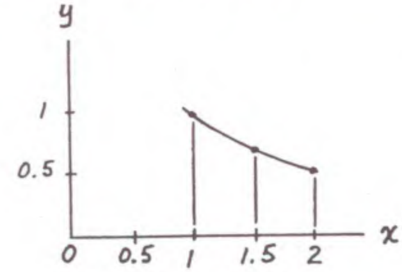
$$-0.122 = \frac{25^2 + 68.49^2 + 19.53^2 - c^2}{2(68.49)19.53}$$

$$= \frac{625 + 4691 + 381 - c^2}{2675}$$

$c^2 = 6023$ $\quad$ $\underline{\underline{c = 77.6\text{ mm}}}$



14-6

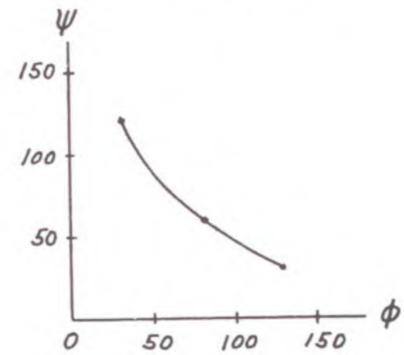


$y = \frac{1}{x}$ from $x = 1$ to $x = 2$

$\phi_s = 30°$, $\Delta\phi = 100°$, $\psi_s = 120°$, $\Delta\psi = 90°$

$$\phi = \phi_s + \frac{\phi_f - \phi_s}{x_f - x_s}(x - x_s)$$

$$= 30 + \frac{100}{1}(x-1) = 30 + 100(x-1)$$



As x increases, y decreases.

Hence as ϕ increases, ψ decreases.

$$\psi = \psi_s + \frac{\psi_f - \psi_s}{y_f - y_s}(y - y_s)$$

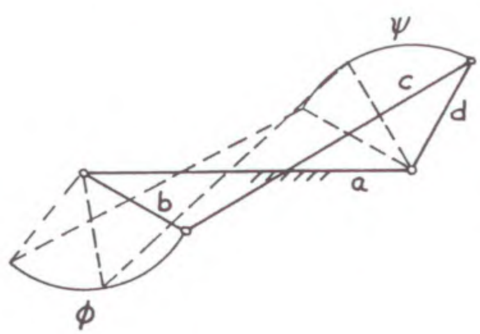
$$= 120 + \frac{30-120}{0.5-1}(y-1) = 120 + 180(y-1)$$

x	y	ϕ, deg	ψ, deg
1	1	30	120
1.5	0.667	80	60
2	0.500	130	30

$$R_1 \cos 30 - R_2 \cos 120 + R_3 = \cos(-90)$$
$$R_1 \cos 80 - R_2 \cos 60 + R_3 = \cos 20$$
$$R_1 \cos 130 - R_2 \cos 30 + R_3 = \cos 100$$

$$0.8660 R_1 + 0.5000 R_2 + R_3 = 0$$
$$0.1736 R_1 - 0.5000 R_2 + R_3 = 0.9397$$
$$0.6428 R_1 - 0.8660 R_2 + R_3 = -0.1736$$

$R_1 = 2.5881$, $R_2 = -2.7317$, $R_3 = -0.8755$



Let $a = 100$

$$b = \frac{a}{R_2} = \frac{100}{-2.7317} = \underline{\underline{-36.61 \text{ mm}}}$$

$$d = \frac{a}{R_1} = \frac{100}{2.5881} = \underline{\underline{38.64 \text{ mm}}}$$

$$R_3 = \frac{a^2 + b^2 + d^2 - c^2}{2bd}$$

$$-0.8755 = \frac{10\,000 + 1340 + 1493 - c^2}{-2829}$$

$$2477 = 12\,833 - c^2$$

$$c^2 = 10\,356, \quad \underline{\underline{c = 101.8 \text{ mm}}}$$

14-7

$$y = \log_e x$$

$$\phi = 0 + \frac{100-0}{2-1}(x-1) = 100(x-1) \qquad (1)$$

$$\psi = 0 + \frac{150-0}{0.6931-0}(y-0) = 216.4\,y \qquad (2)$$

From Eq. (1) $\quad x = \dfrac{\phi}{100} + 1 \qquad (3)$

" " (2) $\quad y = \dfrac{\psi}{216.4} \qquad (4)$

Substitute Eqs. (3) and (4) into the original equation.

$$\frac{\psi}{216.4} = \log_e\left(\frac{\phi}{100}+1\right), \qquad \psi = 216.4\log_e\left(\frac{\phi}{100}+1\right)$$

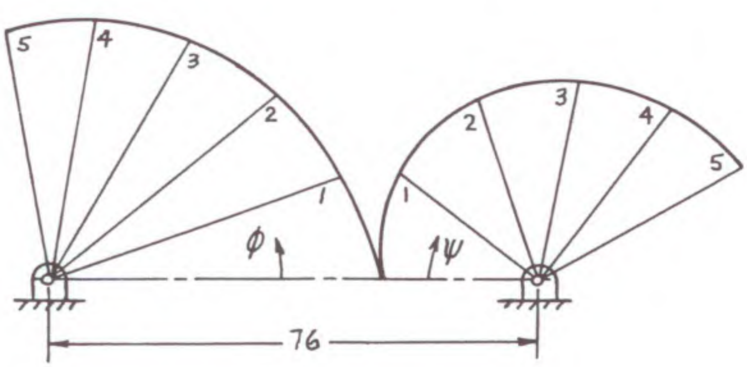
$$\frac{d\psi}{d\phi} = 216.4\,\frac{\frac{1}{100}}{\frac{\phi}{100}+1} = \frac{216.4}{\phi+100}$$

$$r = \frac{O_2O_3}{1+\frac{d\psi}{d\phi}}$$

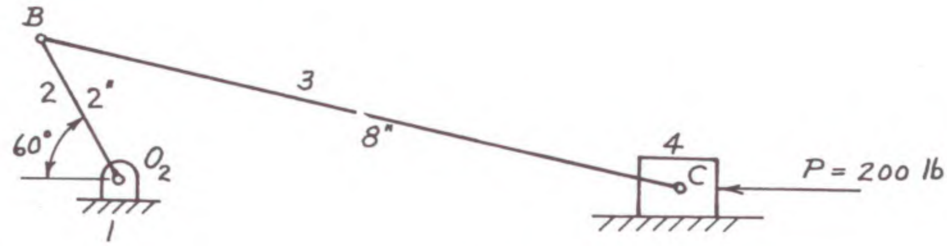
For position 0, $r = \dfrac{76}{1+2.164} = 24.02$ mm

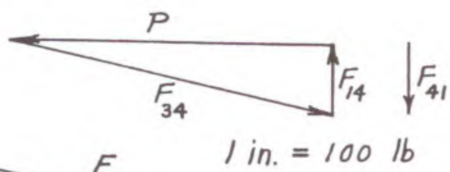
$R = O_2O_3 - r = 76 - 24.02 = 51.98$ mm

Position	x	y	ϕ	ψ	$d\psi/d\phi$	r	R
0	1.0	0	0	0	2.164	24.02	51.98
1	1.2	0.1823	20	39.45	1.803	27.11	48.89
2	1.4	0.3365	40	72.82	1.546	29.85	46.15
3	1.6	0.4700	60	101.71	1.353	32.30	43.70
4	1.8	0.5878	80	127.20	1.202	34.51	41.49
5	2.0	0.6931	100	150.00	1.082	36.50	39.50



16-1

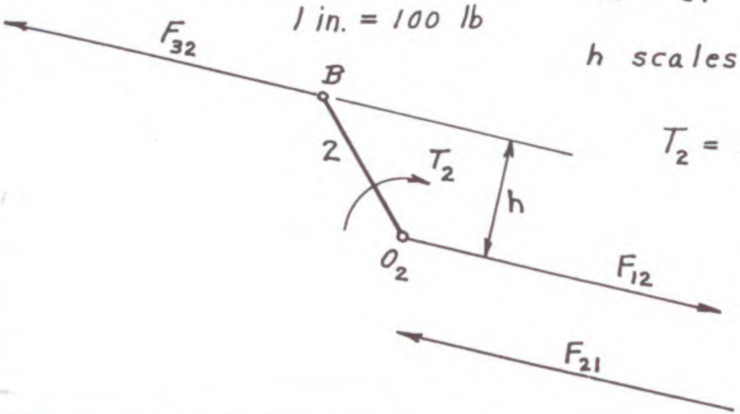




$F_{41} = F_{14}$ scales $\underline{\underline{45 \text{ lb}}}$

$F_{32} = F_{34}$ scales 205 lb

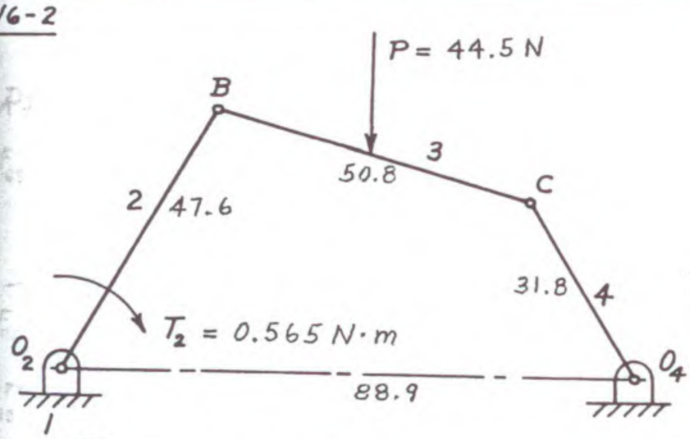
h scales 1.5 in.

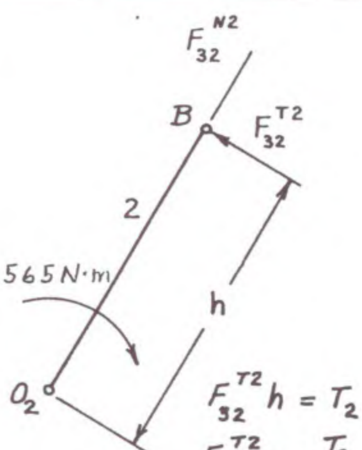


$T_2 = F_{32}h = 205(1.5) = \underline{308 \text{ in.-lb cw}}$

$F_{21} = F_{12} = F_{32} = \underline{\underline{205 \text{ lb}}}$

16-2

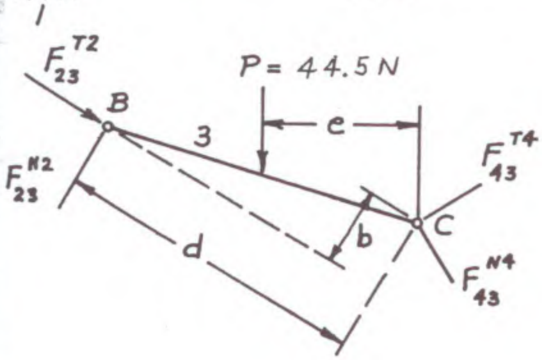




$F_{32}^{T2} h = T_2$

$F_{32}^{T2} = \dfrac{T_2}{h}$

$F_{32}^{T2} = \dfrac{0.565}{0.0476} = 11.9 \text{ N}$



b scales 12.2 mm
d " 49.3 mm
e " 24.4 mm

16-2 (CONT.)

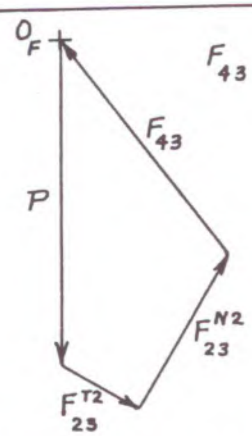
$\Sigma M_c = 0:$

$$F_{23}^{T2}\, b + F_{23}^{N2}\, d + P(24.4) = 0$$

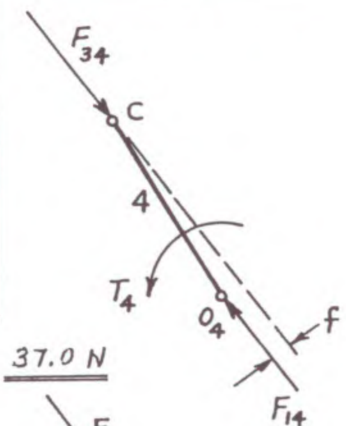
$$119(12.2) + F_{23}^{N2}(49.3) + 1086$$

$$F_{23}^{N2} = \frac{-[11.9(12.2)] - 1086}{49.3} = -25.0 \text{ N}$$

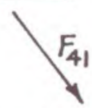
F_{43} scales 37.0 N



Scale : 1mm = 0.876 N



$F_{43} = F_{34} = F_{14} = F_{41} = \underline{37.0 \text{ N}}$



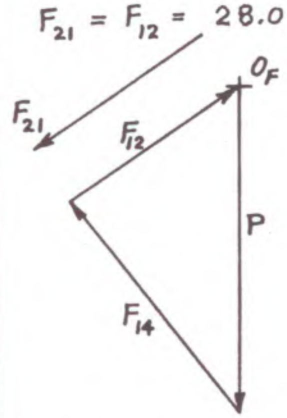
f scales 3.3 mm

$\Sigma M_{O_4} = 0:$

$T_4 = F_{34} f = 37.0(0.0033) = \underline{\underline{0.122 \text{ N}\cdot\text{m}}}$

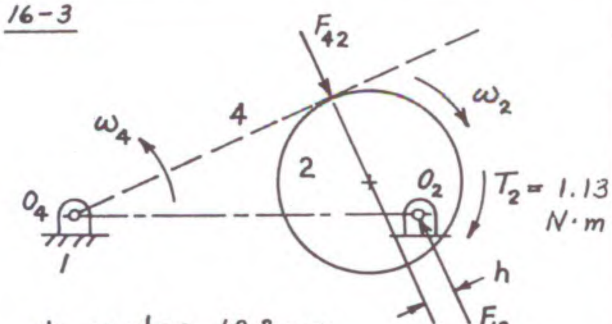
F_{12} scales 28.0 N

$F_{21} = F_{12} = 28.0$ N



Scale : 1 mm = 0.876 N

16-3



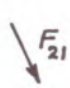
h scales 10.9 mm

$F_{42} h = T_2$, $F_{42} = \dfrac{T_2}{h} = \dfrac{1.13}{0.0109}$

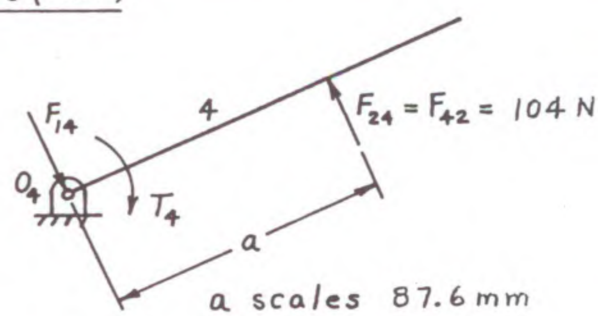
$= 104$ N

$F_{12} = F_{42} = 104$ N

$\underline{F_{21} = F_{12} = 104 \text{ N}}$



16-3 (CONT.)



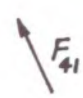
a scales 87.6 mm

$T_4 = F_{24}(a) = 104(0.0876)$

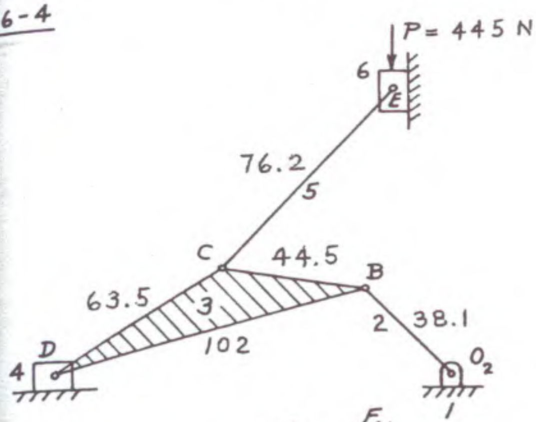
$= 9.11$ N·m cw

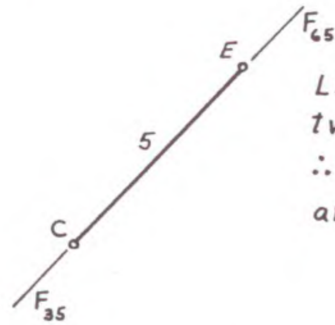
$F_{14} = F_{24} = 104$ N

$\underline{F_{41} = F_{14} = 104 \text{ N}}$

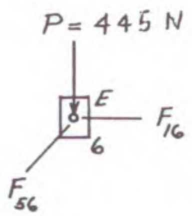


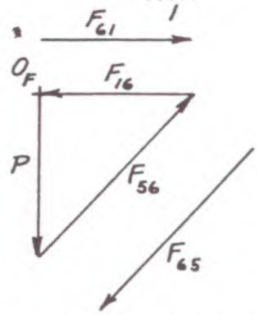
16-4





Link 5 is a two force body. $\therefore F_{35}$ and F_{65} are colinear.





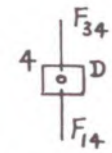
F_{16} scales 409 N

$\underline{F_{61} = F_{16} = 409\ N}$

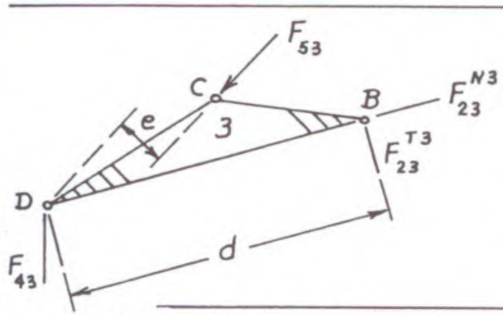
F_{56} scales 614 N

$F_{65} = F_{56} = 614\ N$

Scale : 1 mm = 17.5 N



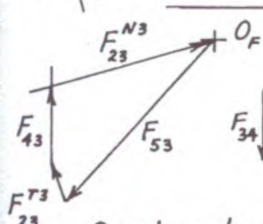
Link 4 is a two force body. $\therefore F_{14}$ and F_{34} are colinear.

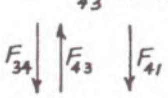


$\Sigma M_D = 0 : \; , \; F_{53}\, e = F_{23}^{T3}\, d$

e scales 15.2 mm

$$F_{23}^{T3} = \frac{F_{53}\, e}{d} = \frac{614\,(15.2)}{102} = 91.5\ N$$



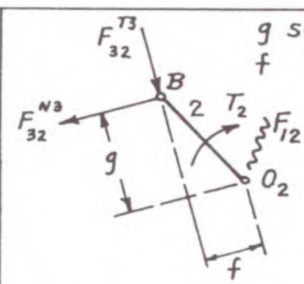


F_{43} scales 205 N

F_{23}^{N3} scales 467 N

$\underline{F_{41} = F_{34} = 205\ N}$

Scale : 1 mm = 17.5 N



g scales 32.5 mm

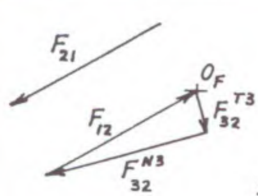
f " 19.1 mm

$\Sigma M_{O_2} = 0 :$

$$T_2 = F_{32}^{T3} f + F_{32}^{N3} g$$

$$= 91.5\,(0.0191) + 467\,(0.0325)$$

$$= \underline{16.9\ N \cdot m\ \text{cw}}$$

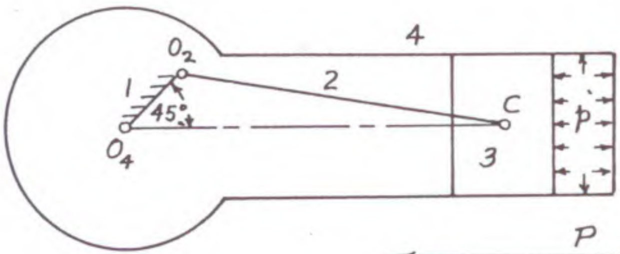


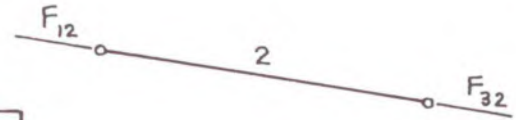
F_{12} scales 480 N

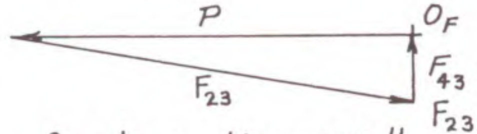
$\underline{F_{21} = F_{12} = 480\ N}$

Scale : 1 mm = 17.5 N

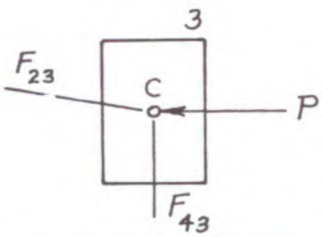
16-5







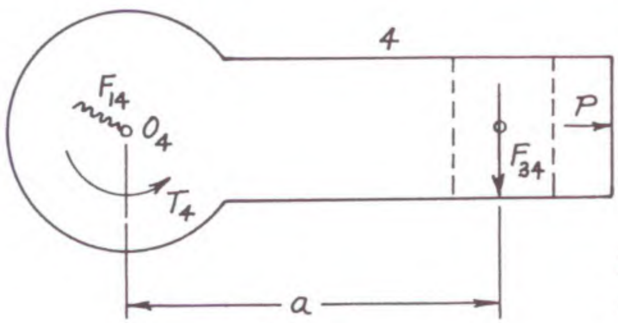
F_{43} scales 42 lb
F_{23} " 250 lb

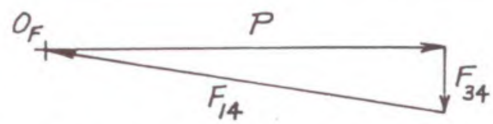


Scale : 1 in. = 100 lb

Piston area, $A = \dfrac{\pi d^2}{4} = \dfrac{\pi (2.5)^2}{4} = 4.9 \text{ in.}^2$

$P = pA = 50\,(4.9) = 245 \text{ lb}$





Scale : 1 in. = 100 lb

a scales 4.62 in.

$\Sigma M_{O_4} = 0:$

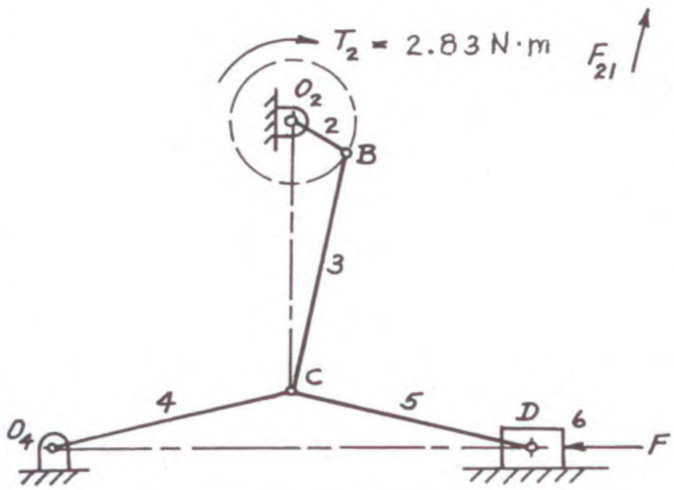
$T_4 = F_{34}\,(a) = 42(4.62)$

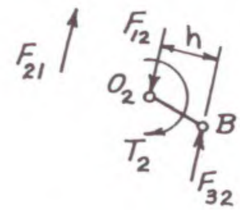
$= \underline{194 \text{ in.-lb ccw}}$

$\underline{F_{21} = F_{12} = F_{32} = F_{23} = 250 \text{ lb}}$

$\underline{F_{41} = F_{14} = 250 \text{ lb}}$

16-6





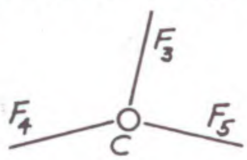
h scales 18.3 mm

$$F_{32}\,h = T_2$$

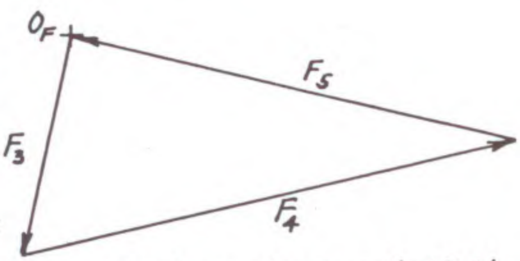
$$F_{32} = \frac{T_2}{h} = \frac{2.83}{0.0183} = 155\ N$$

$$F_{21} = F_{12} = F_{32} = \underline{\underline{155\ N}}$$

Forces on pin at C:



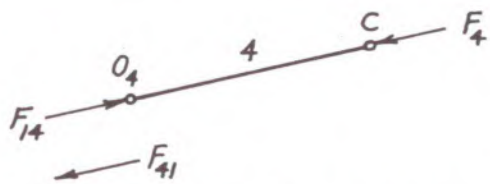
$F_3 = F_{23} = 155\ N$



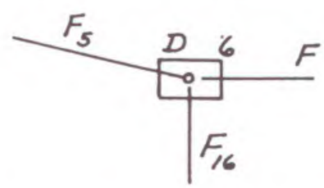
Scale : 1 mm = 4.38 N

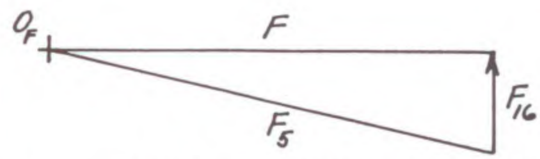
F_4 scales 348 N

F_5 scales 310 N



$$\underline{\underline{F_{41} = F_{14} = F_4 = 348\ N}}$$





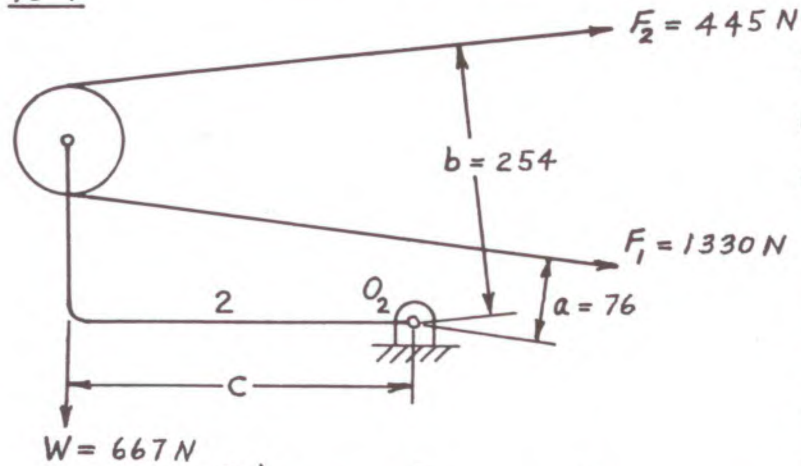
Scale: 1 mm = 4.38 N

$\underline{\underline{F \text{ scales } 300\ N}}$

F_{16} scales 68.9 N

$$\underline{\underline{F_{61} = F_{16} = 68.9\ N}}$$

16-7



$\Sigma M_{O_2} = 0:$

$$667c - 445(254) - 1330(76) = 0$$

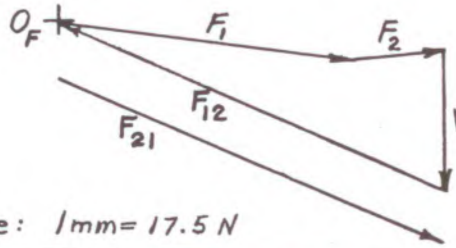
$$c = \frac{113\,030 - 101\,080}{667} = \underline{\underline{321\,mm}}$$

Σ of forces on 2:

$$F_1 \mathbin{+\!\!\!\rightarrow} F_2 \mathbin{+\!\!\!\rightarrow} W \mathbin{+\!\!\!\rightarrow} F_{12} = 0$$

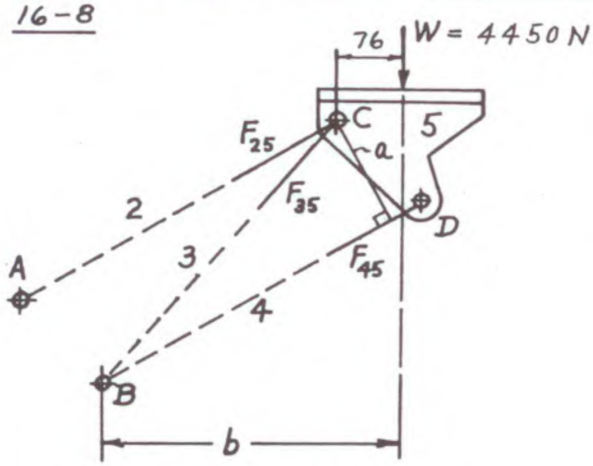
F_{12} scales 1930 N

$\underline{F_{21} = F_{12} = 1930\,N}$



Scale: 1mm= 17.5 N

16-8



Scale : 1 mm = 8 mm

a scales 138 mm

b " 370 mm

Link 5 as a free body:

$\Sigma M_c = 0:$

$$-4450(76) + F_{45}(a) = 0$$

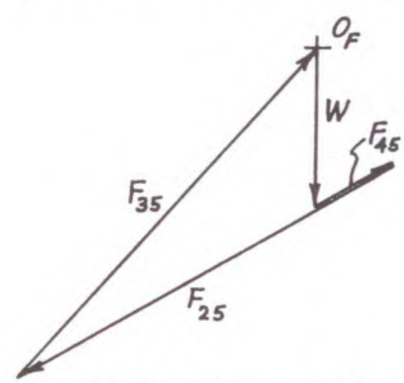
$$F_{45} = \frac{338\,200}{a} = \frac{338\,200}{138} = 2451\,N$$

Links 3, 4 and 5 combined as a free body: $\Sigma M_B = 0:$

$$F_{25}(a) - 4450(b) = 0$$

$$F_{25} = \frac{4450\ (370)}{138} = \underline{\underline{11\,900\,N\ (tension)}}$$

Σ of forces on link 5 equal zero:

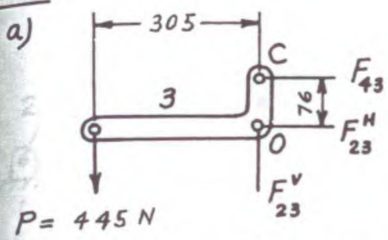


Scale: 1 mm = 175 N

$\underline{F_{35}\ \text{scales}\ 12\,200\,N}$

16-9

a)



$P = 445\ N$

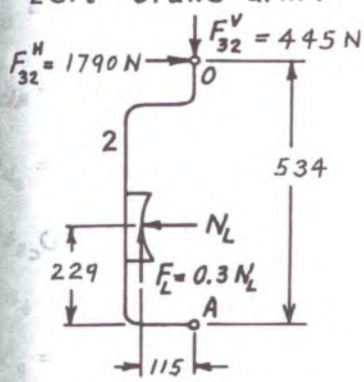
$\Sigma M_o = 0: \quad 76 F_{43} = 445(305)$

$F_{43} = \dfrac{136\,000}{76} = 1790\ N \rightarrow$

$\Sigma F_x = 0: \quad F_{23}^H = F_{43} = 1790\ N \leftarrow$

$\Sigma F_y = 0: \quad F_{23}^V = P = 445\ N \uparrow$

Left brake arm:



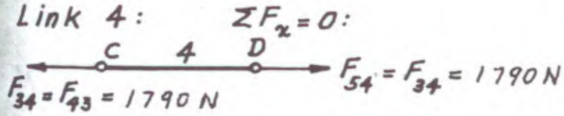
$\Sigma M_A = 0:$

$-1790(534) - 0.3 N_L(115) + 229 = 0$

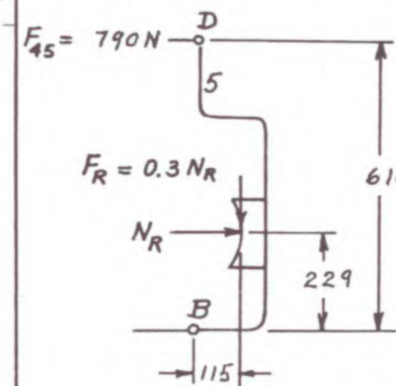
$-956\,000 - 34.5 N_L + 229 N_L = 0$

$N_L = \dfrac{956\,000}{195} = 4900\ N$

Link 4: $\quad \Sigma F_x = 0:$



Right arm



$\Sigma M_B = 0:$

$1790(610) - 229 N_R - 0.3 N_R(115) = 0$

$1\,092\,000 - 229 N_R - 34.5 N_R = 0$

$N_R = \dfrac{1\,092\,000}{264} = 4140\ N$

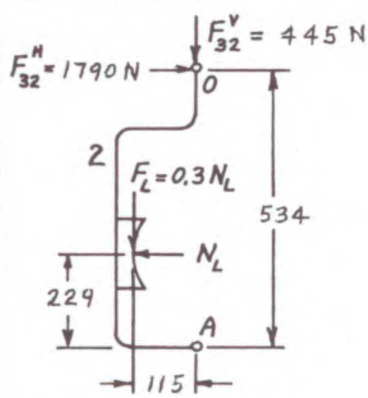
Braking torque:

$T = (F_L + F_R) R \qquad R = \text{drum radius}$

$\quad = f(N_L + N_R) R$

$\quad = 0.3(4900 + 4140)\,0.191 = \underline{\underline{518\ N\cdot m}}$

b) Left arm



$\Sigma M_A = 0: \quad -1790(534) + 229 N_L + 0.3 N_L(115) = 0$

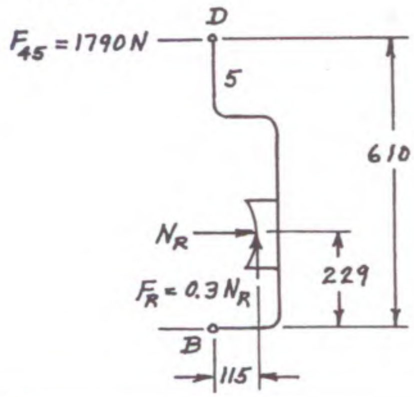
$-956\,000 + 229 N_L + 34.5 N_L = 0$

$N_L = \dfrac{956\,000}{264} = 3620\ N$

CHAPTER 16. STATIC FORCES IN MACHINES

16-9 (CONT.)

Right arm:



$\Sigma M_B = 0:$

$1790(610) + 0.3 N_R (115) - 229 = 0$

$1\,092\,000 + 34.5 N_R - 229 = 0$

$195 N_R = 1\,092\,000, \quad N_R = 5\,600\,N$

Braking torque:

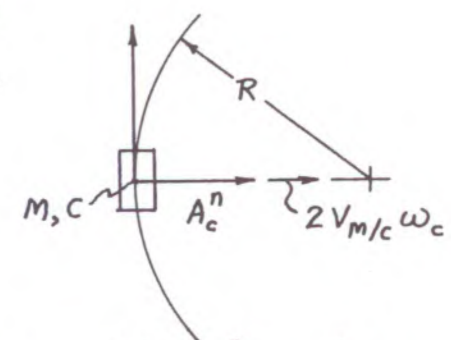
$T = f(N_L + N_R)R$

$= 0.3(3620 + 5600)\,0.191 = \underline{528\ N \cdot m}$

CHAPTER 17. INERTIA FORCES IN MACHINES

17-1

a)



$$\overset{--}{A_M} = \overset{\checkmark\checkmark}{A_c^n} +\!\!\!\rightarrow \overset{o}{A_c^t} +\!\!\!\rightarrow \overset{o}{A_{M/c}^n} +\!\!\!\rightarrow \overset{o}{A_{M/c}^t}$$

$$+\!\!\!\rightarrow 2V_{M/c}\,\omega_c$$

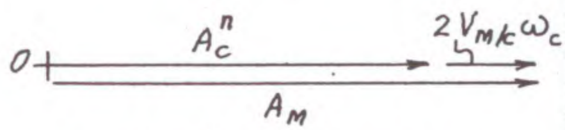
b) $V_c = \dfrac{55 \times 5280}{60 \times 60} = 80.667\ ft/s$

$A_c^n = \dfrac{V_c^2}{R} = \dfrac{80.667^2}{400} = 16.268\ ft/s^2$

$\omega_c = \dfrac{V_c}{R} = \dfrac{80.667}{400} = 0.202\ rad/s\ cw$

$2V_{M/c}\,\omega_c = 2 \times 5 \times 0.202$

$= 2.02\ ft/s^2$



Accel. of man becomes

$A_M = A_c^n +\!\!\!\rightarrow 2V_{M/c}\,\omega_c$

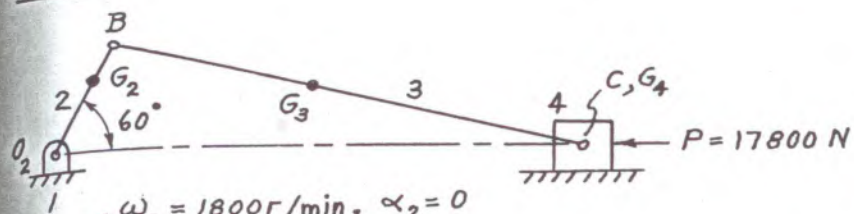
$= 16.268 + 2.02 = \underline{\underline{18.29\ ft/s^2}}$

and is directed radially inward

c) $f = MA = \dfrac{160}{32.2} \times 18.29 = \underline{\underline{90.9\ lb}}$

and is directed radially outward

17-2



$\omega_2 = 1800\ r/min, \quad \alpha_2 = 0$

$\omega_2 = \left(\frac{1800}{60}\right) 2\pi = 188\ rad/s$

$\omega_3 = \frac{V_{C/B}}{BC} = \frac{7.47}{0.286} = 26.1\ rad/s\ \text{cw}$

$\overset{\circ}{A_C^n} \mathrel{+\!\!\to} \overset{-\checkmark}{A_C^t} = \overset{\checkmark\checkmark}{A_B^n} \mathrel{+\!\!\to} \overset{\circ}{A_B^t} \mathrel{+\!\!\to} \overset{\checkmark\checkmark}{A_{C/B}^n} \mathrel{+\!\!\to} \overset{-\checkmark}{A_{C/B}^t}$

$A_B^n = (O_2B)\,\omega_2^2 = 0.0762(188)^2 = 2690\ m/s^2$

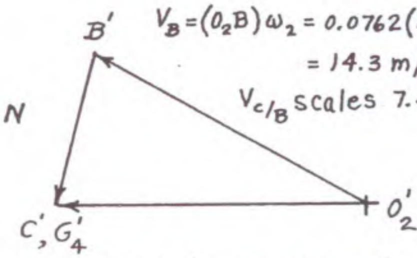
$A_{C/B}^n = \frac{V_{C/B}^2}{BC} = \frac{(7.47)^2}{0.286} = 195\ m/s^2$

$A_{G_2} = \frac{50.8}{76.2} = \frac{50.8}{76.2}(2690) = 1793\ m/s^2$

A_{G_3} scales $1780\ m/s^2$

A_{G_4} scales $1008\ m/s^2$

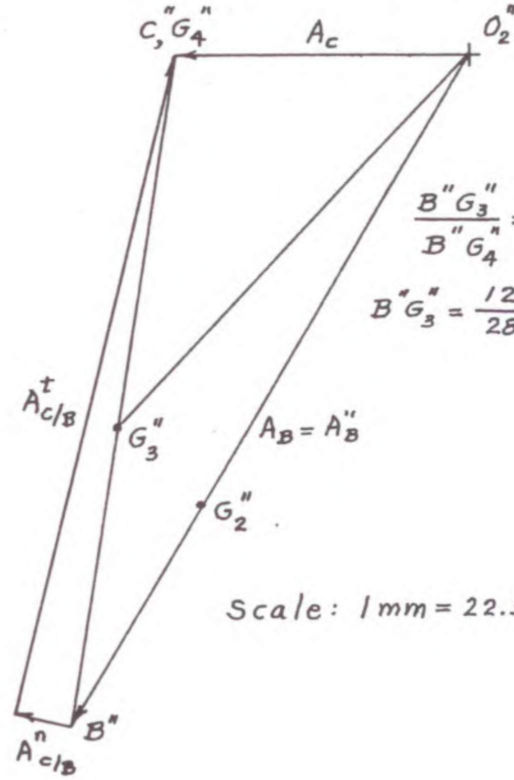
$\alpha_3 = \frac{A_{C/B}^t}{BC} = \frac{2363}{0.286} = 8262\ rad/s^2\ \text{ccw}$



$V_B = (O_2B)\,\omega_2 = 0.0762(188)$
$= 14.3\ m/s$

$V_{C/B}$ scales $7.47\ m/s$

Scale: $1\ mm = 0.3\ m/s$



$\frac{B''G_3''}{B''G_4''} = \frac{BG_3}{BG_4}$

$B''G_3'' = \frac{127}{286}\,B''G_4''$

Scale: $1\ mm = 22.5\ m/s^2$

17-2 (CONT.)

Inertia forces:

$f_2 = M_2 A_{G_2} = 2.26\,(1793) = 4052\ N$

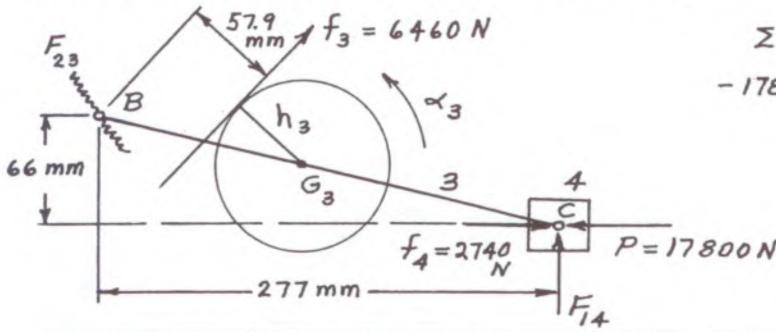
$f_3 = M_3 A_{G_3} = 3.63\,(1780) = 6460\ N$

$f_4 = M_4 A_{G_4} = 2.72\,(1008) = 2740\ N$

$fh = I\alpha, \quad h_3 = \dfrac{I_3 \alpha_3}{f_3} = \dfrac{0.0408(8262)}{6460}$

$= 0.0522\ m = 52.2\ mm$

Bodies 3 and 4 combined as a free body:



$\Sigma M_B = 0:$

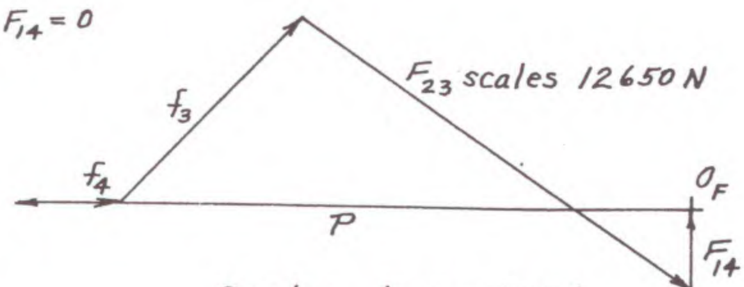
$-17800(66) + 2740(66) + F_{14}(277)$

$+ 6460(57.9) = 0$

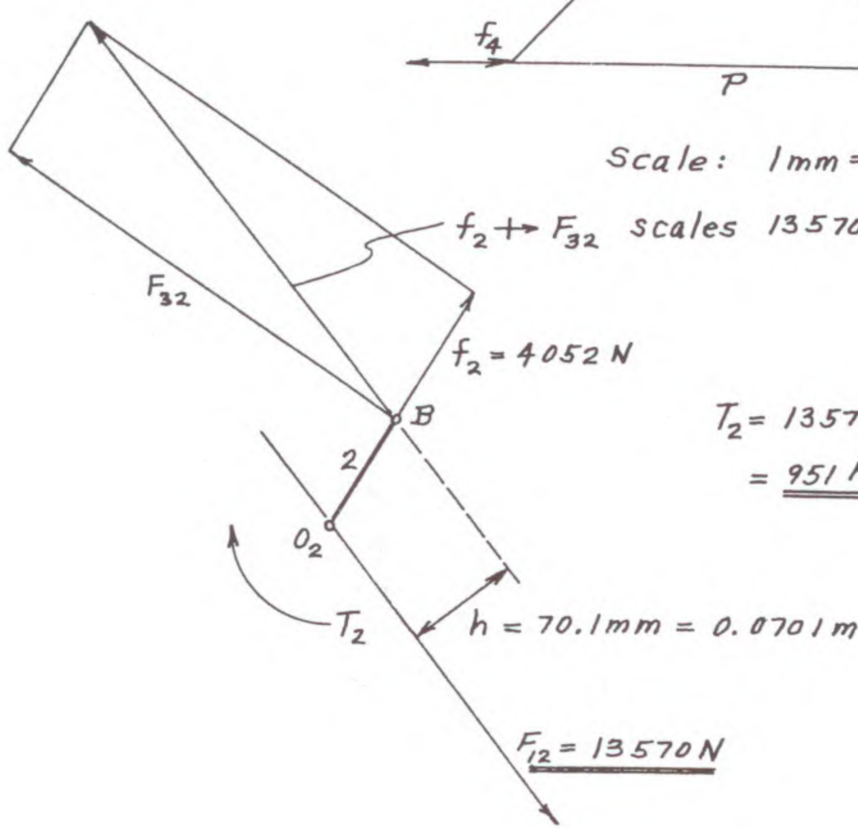
$F_{14} = \dfrac{620\,000}{277} = \underline{\underline{2238\ N\uparrow}}$

For equilib. Σ forces $= 0$:

$P \mathrel{+\!\!\rightarrow} f_4 \mathrel{+\!\!\rightarrow} f_3 \mathrel{+\!\!\rightarrow} F_{23} \mathrel{+\!\!\rightarrow} F_{14} = 0$



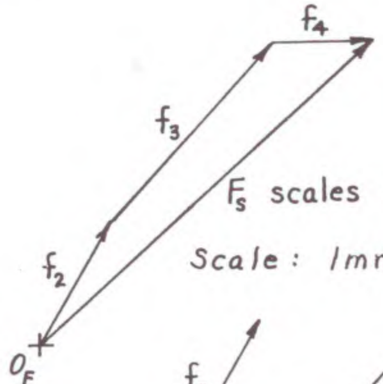
Scale: 1mm = 175 N



$T_2 = 13570(0.0701)$

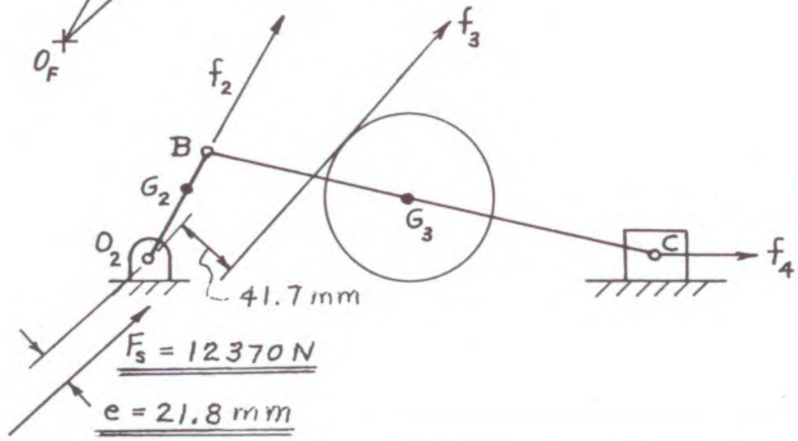
$= \underline{\underline{951\ N \cdot m\ CW}}$

17-2 (CONTIN.)



F_s scales 12370 N

Scale: 1 mm = 175 N



$\underline{\underline{F_s = 12370\ N}}$

$\underline{\underline{e = 21.8\ mm}}$

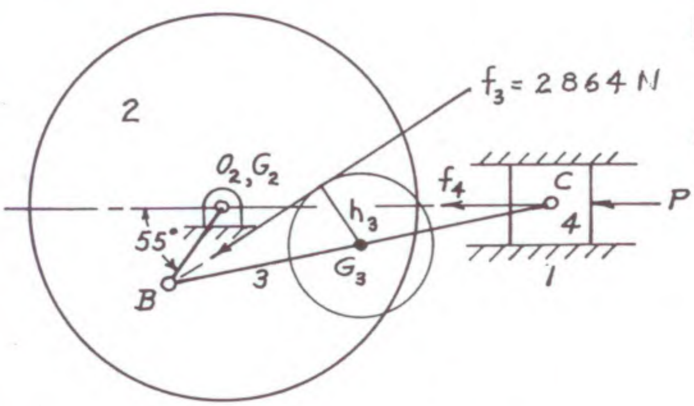
ΣM_{O_2}:

$$F_s e = f_2(0) + f_3(41.7) + f_4(0)$$

$$e = \frac{6460\ (41.7)}{F_s}$$

$$= \frac{6460(41.7)}{12\,370} = 21.8\ mm$$

17-3

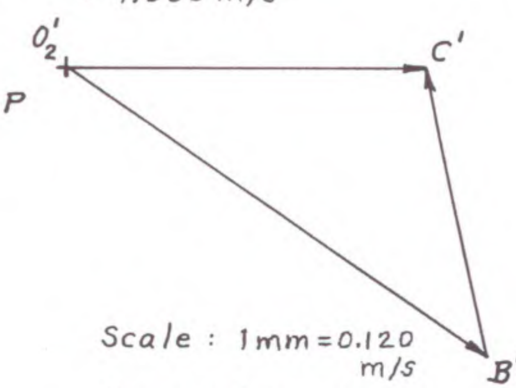


$$\omega_3 = \frac{V_{C/B}}{BC} = \frac{5.55}{0.610} = 9.1\ rad/s\ ccw$$

Velocities:

$$V_B = \frac{2\pi R n}{60} = 2\pi\ \frac{0.152}{60}\ (600)$$

$$= 9.550\ m/s$$



Scale: 1 mm = 0.120 m/s

$V_{C/B}$ scales 5.55 m/s

7-3 (CONTIN.)

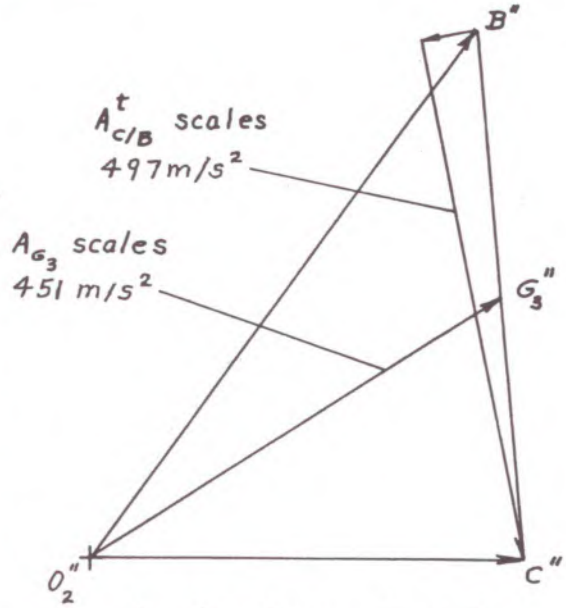
Accelerations:

$$A_C^n \nrightarrow A_C^t = A_B^n \nrightarrow A_B^t \nrightarrow A_{C/B}^n \nrightarrow A_{C/B}^t$$

$$A_B^n = \frac{V_B^2}{O_2B} = \frac{9.55^2}{0.152} = 600\ m/s^2$$

$$A_{C/B}^n = \frac{V_{C/B}^2}{BC} = \frac{5.55^2}{0.610} = 50.5\ m/s^2$$

$$\alpha_3 = \frac{A_{C/B}^t}{BC} = \frac{497}{0.610} = 815\ rad/s^2 cw$$



Scale: 1 mm = 6 m/s²

Force on 4 due to air pressure:

$$P = p\,\pi\,\frac{D^2}{4} = 209\,000\,\pi\,\frac{0.127^2}{4} = 2650\ N$$

$$f_3 = M_3 A_{G_3} = 6.35(451) = 2864\ N$$

$$f_4 = M_4 A_{G_4} = 3.63(396) = 1437\ N$$

$$h_3 = \frac{I_3 \alpha_3}{f_3} = \frac{0.394(815)}{2864} = 0.112\ m = 112\ mm$$

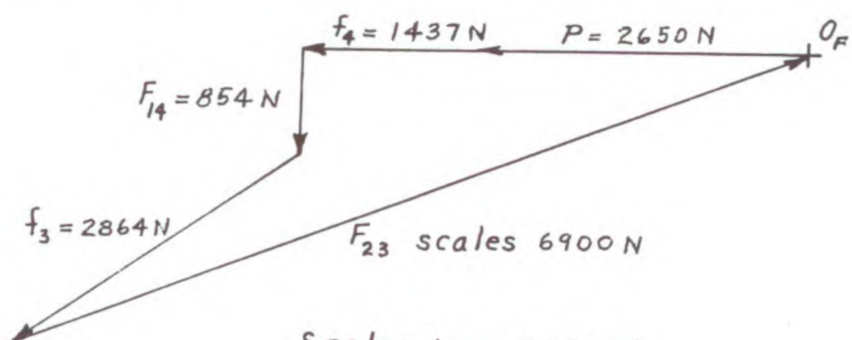
Bodies 3 and 4 combined as a free body -

$\Sigma M_B = 0:$ $\quad 2864(0) - F_{14}\ (594) + (2650 + 1437)124 = 0$

F_{14} assumed acting downward.

$$F_{14} = \frac{507\,000}{594} = 854\ N \downarrow$$

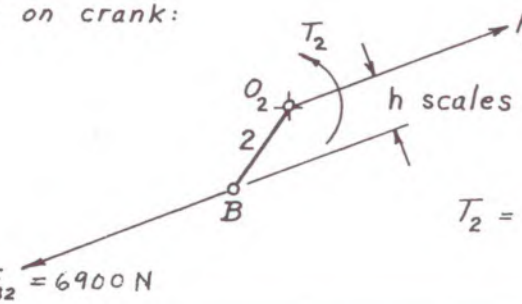
Σ forces $= 0:$ $\quad P \nrightarrow f_4 \nrightarrow F_{14} \nrightarrow f_3 \nrightarrow F_{23} = 0$



Scale: 1 mm = 52.5 N

17-3 (CONTIN.)

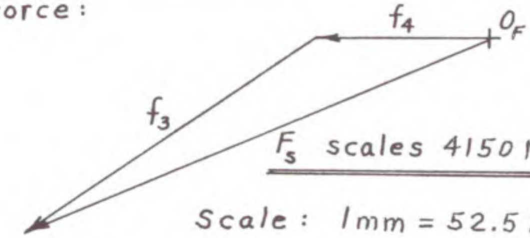
Forces on crank:



h scales 86.9 mm = 0.0869 m

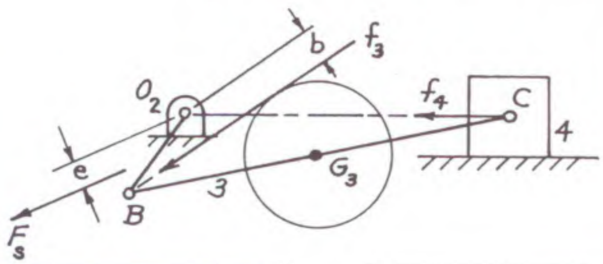
$T_2 = F_{32}\,h = 6900(0.0869) = \underline{\underline{600\ \text{N}\cdot\text{m ccw}}}$

Shaking force:



$\underline{\underline{F_s \text{ scales } 4150\ \text{N}}}$

Scale: 1 mm = 52.5 N



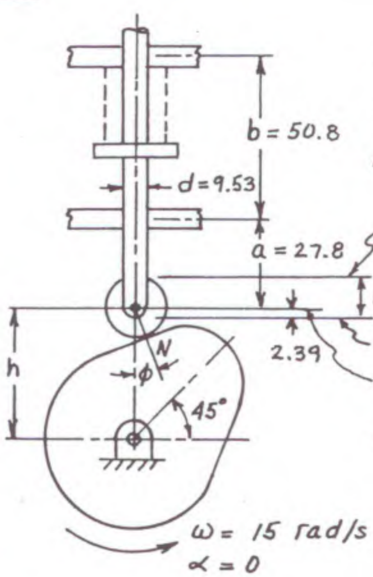
$\Sigma M_B = 0:$

$F_s\,(e) = f_3(53.3) + f_4(0)$

$4150\,e = 2864(53.3)$

$e = \dfrac{153\,000}{4150} = \underline{\underline{36.9\ \text{mm}}}$

17-4



F_s = spring force

$k = 3.5$ N/mm

$\mu = 0.1$

Highest position, $F_s = 17.8 + 12.7(3.5) = 62.3$ N

Lowest position, $F_s = 17.8$ N

At this position, $F_s = 17.8 + 2.39(3.5) = 26.2$ N

ϕ measures 19°, $\sin\phi = 0.3256$

Follower vel. = 0.219 m/s ↑

" accel. = 11.2 m/s^2 ↑

17-4 (CONT.)

a) $f = MA = 1(11.2) = 11.2\text{ N}$

$$F = P + f + W + F_s$$

$$= 22.2 + 11.2 + 9.81 + 26.2 = 69.41\text{ N}\downarrow$$

$$N = \frac{Fb}{b\cos\phi - (2\mu a + \mu b - \mu^2 d)\sin\phi}$$

$$= \frac{69.4(50.8)}{50.8(0.9455) - [(2\times 0.1\times 27.8) + (0.1\times 50.8) - (0.01\times 9.53)]0.3256}$$

$$= \frac{3526}{48 - 3.43} = \underline{\underline{79.1\text{ N}}}$$

b) $$\tan\phi_m = \frac{b}{\mu(2a + b - \mu d)} = \frac{50.8}{0.1[(55.6 + 50.8) - (0.1\times 9.53)]}$$

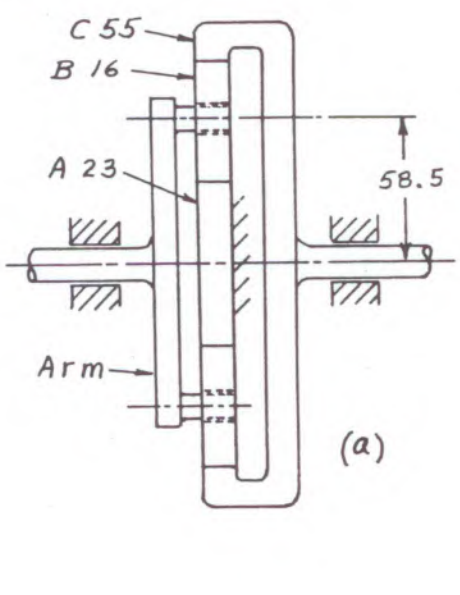
$$= \frac{50.8}{0.1(105)} = 4.84, \qquad \underline{\underline{\phi_m = 78.3^\circ}}$$

c) $T = Nh\sin\phi$, but $h = 28.6 + 9.53 + 2.39 = 40.5\text{ mm}$

$$= 79.1(0.0405)0.3256$$

$$= \underline{\underline{1.04\text{ N}\cdot\text{m ccw}}}$$

17-5



(a)

Pitch diameters:

$D = mN$, $D_A = 3(23) = 69\text{ mm}$

$D_B = 3(16) = 48\text{ mm}$

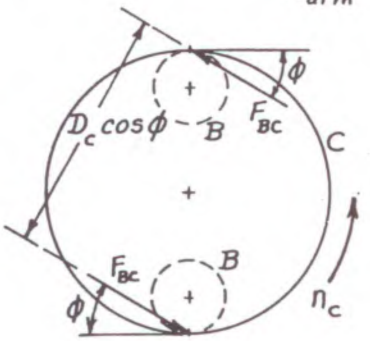
$D_C = 3(55) = 165\text{ mm}$

17-5 (CONT.)

Member	Arm	A	B	C
Train locked and arm given one positive turn	+1	+1	+1	+1
Arm fixed, A given one negative turn	0	−1	$+\frac{23}{16}$	$+\frac{23}{16}\times\frac{16}{55}$
Resultant turns	+1	0	+2.438	+1.418

Driver (Arm)   Driven (C)

From table $\dfrac{n_c}{n_{arm}} = 1.418$, $\quad n_c = 1.418\, n_{arm} = 1.418(2000) = \underline{\underline{2836 \text{ r/min CCW}}}$



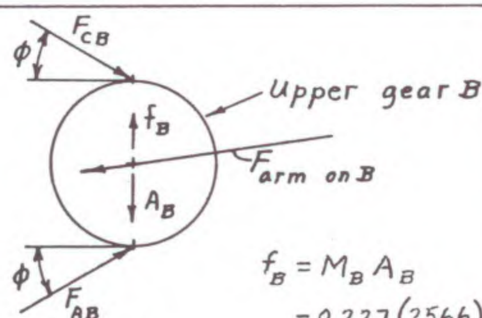
(b)

Torque driven shaft exerts on C:

$$T = \frac{9.55\,W}{n} = \frac{9.55(37300)}{2836} = \underline{\underline{126\ \text{N·m}}}$$

$$F_{BC}\,(D_c\cos\phi) = 126\ \text{N·m}$$

$$F_{BC} = \frac{126}{D_c\cos\phi} = \frac{126}{0.165(0.9397)} = \underline{\underline{813\ \text{N}}}$$



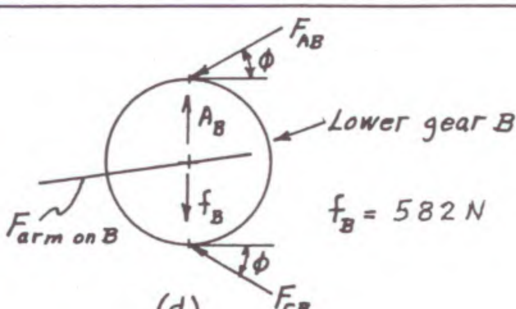
(c)

$f_B = M_B A_B$
$= 0.227(2566)$
$= 582$ N

$F_{CB} = F_{BC} = \underline{\underline{813\ \text{N}}}$

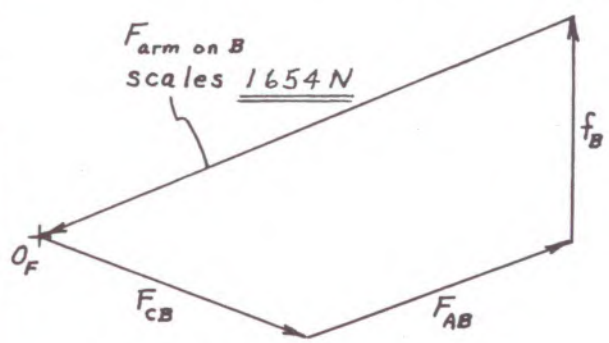
Since torque in B must equal zero,
$\underline{\underline{F_{AB} = 813\ \text{N}}}$

$$A_B = R\omega_{arm}^2 = 0.0585\left(\frac{2000\times 2\pi}{60}\right)^2 = 2566\ \text{m/s}^2$$



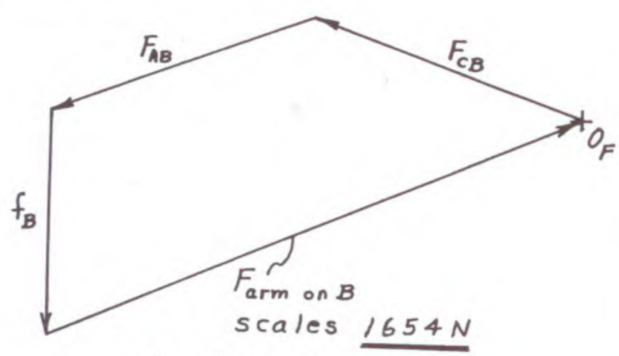
(d)

$f_B = 582$ N



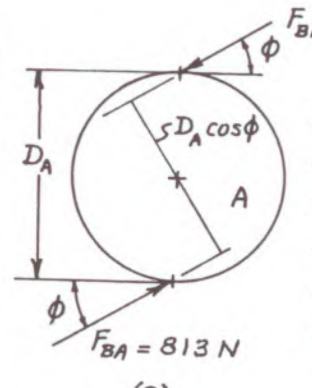
Scale: 1 mm = 18 N

17-5 (CONT.)



scales $\underline{1654\ N}$

Scale: 1 mm = 18 N



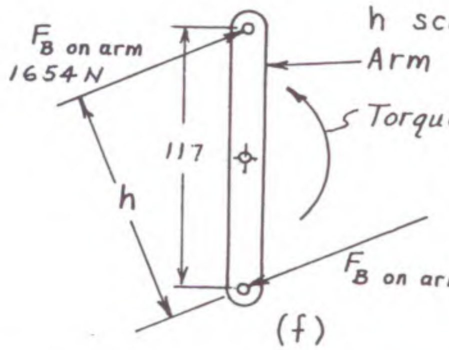
(e)

Resisting torque frame exerts on A is

$F_{BA}\,(D_A \cos\phi)$

$= 813\,(0.069 \times 0.9397)$

$= \underline{\underline{52.7\ N\cdot m\ cw}}$



(f)

h scales 110 mm

Arm

Torque driving shaft exerts on arm $= 1654\,h$

$= 1654\,(0.110)$

$= \underline{\underline{182\ N\cdot m\ ccw}}$

17-6

$$T = \frac{198.5}{100} = 1.985\ s$$

$$I = Mr\left[\left(\frac{T}{2\pi}\right)^2 g - r\right]$$

$$= 40(0.025)\left[\left(\frac{1.985}{2\pi}\right)^2 9.81 - 0.025\right]$$

$$= 1\left[(0.316)^2 9.81 - 0.025\right]$$

$$= 1\left[0.980 - 0.025\right]$$

$$= \underline{\underline{0.955\ kg\cdot m^2}}$$

17-7

a) $T = \dfrac{189}{200} = 0.945\ s$

$$I = Mr\left[\left(\frac{T}{2\pi}\right)^2 g - r\right]$$

$$= 1.02(0.178)\left[\left(\frac{0.945}{2\pi}\right)^2 9.81 - 0.178\right]$$

17-7 (CONT.)

$$I = 0.182\left[0.222 - 0.178\right]$$

$$= \underline{\underline{0.00800\ kg\cdot m^2}}$$

b) $M_1 + M_2 = M = 1.02 \qquad (1)$

$M_1 h_1 = M_2 h_2$

$M_1(0.168) = M_2 h_2 \qquad (2)$

Substitute Eq.(2) into (1).

$$\frac{M_2 h_2}{0.168} + M_2 = 1.02$$

or

$$M_2 = \frac{1.02}{1 + \dfrac{h_2}{0.168}} \qquad (3)$$

17-7 (CONT.)

$$M_1 h_1^2 + M_2 h_2^2 = I$$

or
$$M_2 = \frac{I - M_1 h_1^2}{h_2^2}$$

$$= \frac{0.00800 - \frac{M_2 h_2}{0.168}(0.0282)}{h_2^2}$$

$$= \frac{0.00800 - 0.168\, M_2 h_2}{h_2^2}$$

$$M_2 h_2^2 = 0.00800 - 0.168\, M_2 h_2$$

$$M_2\left(h_2^2 + 0.168\, h_2\right) = 0.00800$$

$$M_2 = \frac{0.00800}{h_2^2 + 0.168\, h_2} \qquad (4)$$

Equate Eqs. (3) and (4).

$$\frac{1.02}{1 + 5.95\, h_2} = \frac{0.00800}{h_2^2 + 0.168\, h_2}$$

Cross multiply.

$$1.02\, h_2^2 + 0.171\, h_2 = 0.00800 + 0.0476\, h_2$$

$$1.02\, h_2^2 + 0.123\, h_2 - 0.00800 = 0$$

$$h_2^2 + 0.121\, h_2 - 0.00800 = 0$$

$$h_2 = \frac{-0.121 \pm \sqrt{0.0146 + 0.0320}}{2}$$

$$= \frac{-0.121 \pm 0.216}{2} = \frac{0.095}{2} = 0.0475 \text{ m}$$

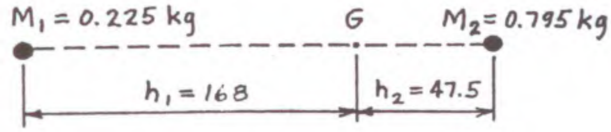
$$= \underline{\underline{47.5 \text{ mm}}}$$

Substitute into Eq. (3).

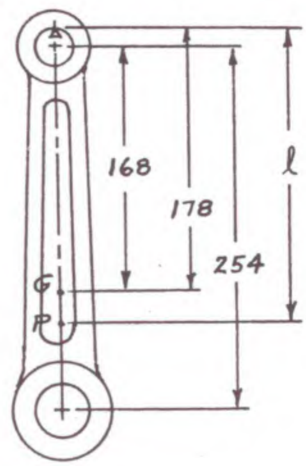
$$M_2 = \frac{1.02}{1 + \frac{0.0475}{0.168}} = \frac{1.02}{1 + 0.283} = \underline{\underline{0.795 \text{ kg}}}$$

Substitute into Eq. (1).

$$M = 1.02 - M_2 = 1.02 - 0.795 = \underline{\underline{0.225 \text{ kg}}}$$



c)



$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$l = \left(\frac{T}{2\pi}\right)^2 g$$

$$= \left(\frac{0.945}{2\pi}\right)^2 9.81$$

$$= (0.150)^2\, 9.81$$

$$= 0.0226\,(9.81)$$

$$= 0.221 \text{ m}$$

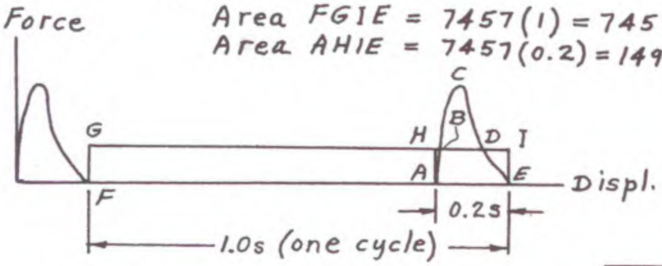
$$= \underline{\underline{221 \text{ mm}}}$$

18-1

a) $7457\ W = 7457\ J/s$

Area $FGIE = 7457(1) = 7457\ J$

Area $AHIE = 7457(0.2) = 1491\ J$ supplied by motor during crushing



Area $ABCDE$ = Area $FGIE = 7457\ J$

Energy supplied by flywheel $= 7457 - 1491 = \underline{5966\ J}$

b) $V = \dfrac{\pi D n}{60} = \dfrac{\pi(1.829)60}{60} = 5.75\ m/s$

$M = \dfrac{E}{CV^2} = \dfrac{5966}{0.2(5.75)^2} = 902\ N\cdot s^2/m$

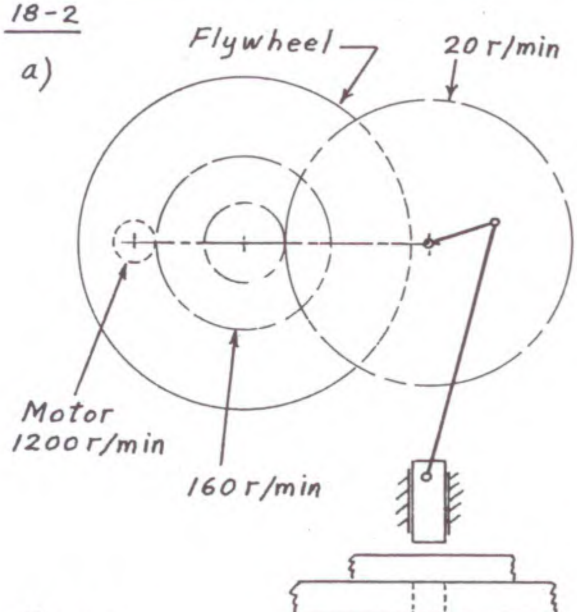
$M = 902\dfrac{N\cdot s^2}{m} = 902\dfrac{kg\cdot m}{s^2}\cdot\dfrac{s^2}{m} = 902\ kg$

Rim mass $= 0.9M$

$= 0.9(902) = \underline{\underline{812\ kg}}$

18-2

a)



Time for one cycle $= \dfrac{60}{20} = 3\ s$

$P = \pi d t \tau = \pi(0.017)(0.019)310\times10^6 = 315\,000\ N$

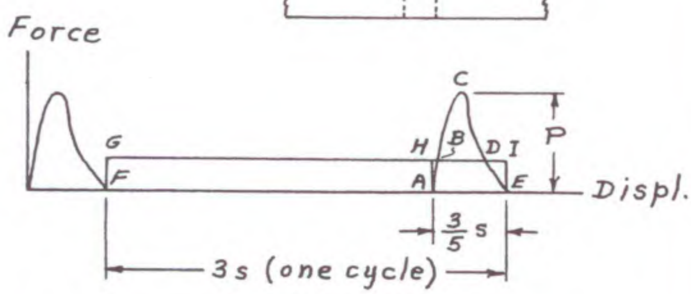
$W_k = \dfrac{1}{2}Pt = \dfrac{1}{2}(315\,000)0.019 = 2993\ J$

Av. power req'd. during actual punching

$= \dfrac{2993}{3/5} = 4988\ J/s = 4988\ W$

No flywheel:

Maximum power req'd $= 2(4988) = \underline{\underline{9976\ W}}$



Energy supplied by flywheel during punching $= 2993 - 599 = 2394\ J$

b) Flywheel used:

Power $= \dfrac{2993}{3} = \underline{\underline{998\ W}}$

c) Energy supplied by motor during 3 s interval = energy required for punching $= 2993\ J$

Energy supplied by motor during $\frac{3}{5}$ s interval

$= \dfrac{2993}{5} = 599\ J$

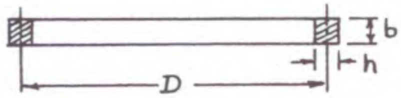
18-2 (CONT.)

$V = 20\ m/s\ ;\ M = \dfrac{E}{CV^2} = \dfrac{2394}{0.1(20)^2} = 59.9\ N\cdot s^2/m = 59.9\ kg\cdot\dfrac{m}{s^2}\cdot\dfrac{s^2}{m} = 59.9\ kg$

Mass of rim $= 0.9M = 0.9(59.9) = \underline{\underline{53.9\ kg}}$

d) Total mass of flywheel $= 1.25(53.9) = \underline{\underline{67.4\ kg}}$ approx.

e)



$V = \pi D \dfrac{n}{60}$

$D = \dfrac{V(60)}{\pi n} = \dfrac{20(60)}{\pi(160)} = 2.39\ m = \underline{\underline{2390\ mm}}$

Rim vol. $= \pi D b h$

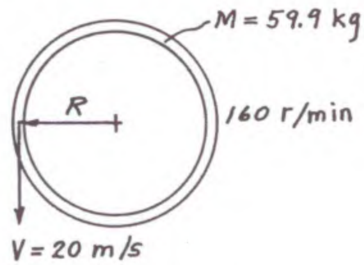
" mass $= \pi D b h$ (density)

$53.9 = \pi(2.390)\, b h\, (7090)$

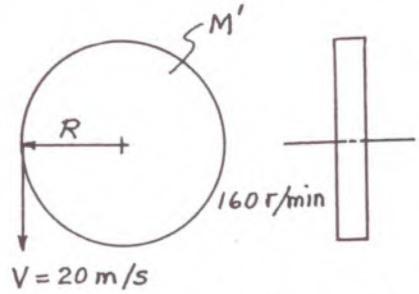
$bh = \dfrac{53.9}{\pi(2.390)\,7090} = 0.001012\ m^2$

But $b = h = \sqrt{0.001012} = 0.0318\ m = \underline{\underline{31.8\ mm}}$

f)



$I = Mk^2 = MR^2$



$I' = M'(k')^2 = M'\left(\dfrac{R}{\sqrt{2}}\right)^2 = M'\dfrac{R^2}{2}$

$K.E. = \dfrac{1}{2} I \omega^2$

For K.E. to be same for both flywheels

then $I = I'$

or $M = \dfrac{M'}{2}$ or $M' = 2M = 2(59.9) = \underline{\underline{119.8\ kg}}$

18-3

$\theta = 720^\circ = 4\pi$ radians

1mm of abscissa $= \dfrac{4\pi}{184}$

$= 0.0683$ radian

1mm of ordinate $= 5.34\ N\cdot m$

Work per square mm of area:

Work $= T\theta$

$= 5.34(0.0683) = 0.365$ joules

Max. energy change between points A and E

$= (-774 + 510 - 793 + 445 - 755)\, 0.365$

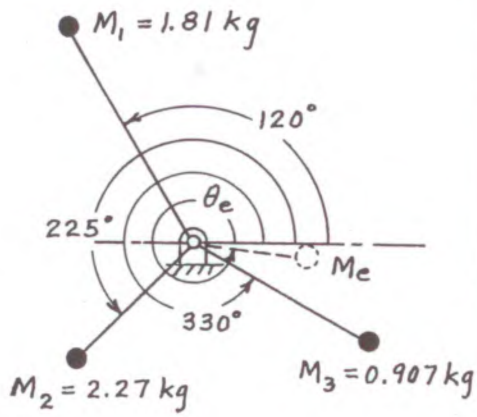
$= (-1367)\, 0.365 = -499$ joules

$V = \pi D n = \pi(0.610)\dfrac{1000}{60} = 31.9\ m/s$

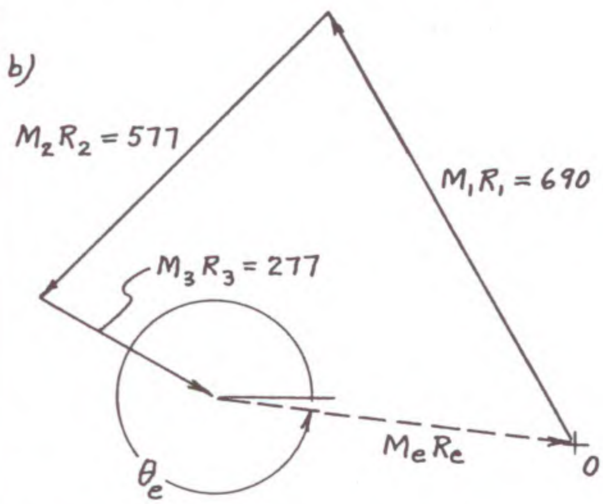
$M = \dfrac{E}{CV^2} = \dfrac{499}{0.05(31.9)^2} = \underline{9.81\ kg}$

19-1

a)



b)



Scale: $1\,mm = 9\,MR$ units

M_eR_e scales 513 units

$$M_e = \frac{513}{R_e} = \frac{513}{178} = \underline{\underline{2.88\ kg}}$$

$\underline{\underline{\theta_e \text{ measures } 354°}}$

No.	M, kg	R, mm	θ, deg	cos θ	sin θ	MR cos θ	MR sin θ
1	1.81	381	120	-0.500	0.866	-345	598
2	2.27	254	225	-0.707	-0.707	-408	-408
3	0.907	305	330	0.866	-0.500	240	-139
						Σ = -513	Σ = 51

$$\Sigma MR\cos\theta + M_eR_e\cos\theta_e = 0$$

$$-513 + M_eR_e\cos\theta_e = 0 \qquad (1)$$

$$\Sigma MR\sin\theta + M_eR_e\sin\theta_e = 0$$

$$51 + M_eR_e\sin\theta_e = 0 \qquad (2)$$

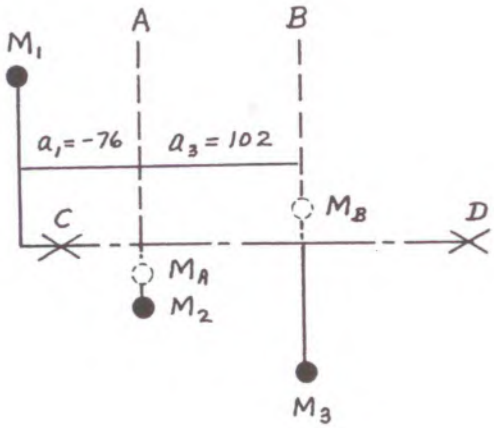
$$\frac{M_eR_e\sin\theta_e}{M_eR_e\cos\theta_e} = \frac{-51}{513}, \quad \tan\theta_e = -0.0994$$

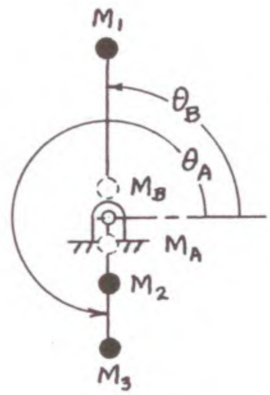
Because $\sin\theta_e$ is (−) and $\cos\theta_e$ is (+), θ_e lies in 4^{th} quadrant

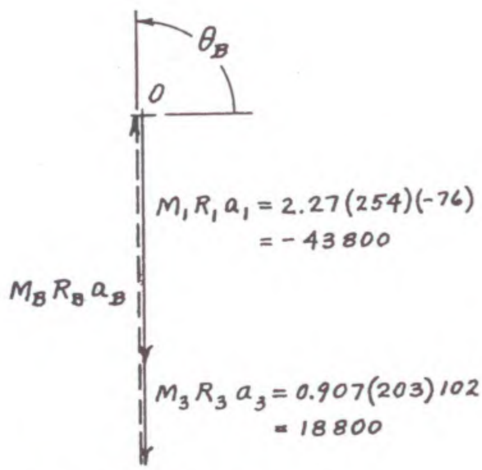
$\underline{\underline{\theta_e = 354°}}$

From Eq. (1), $M_e = \dfrac{513}{R_e\cos\theta_e} = \dfrac{513}{178(0.9945)} = \underline{\underline{2.88\ kg}}$

19-2







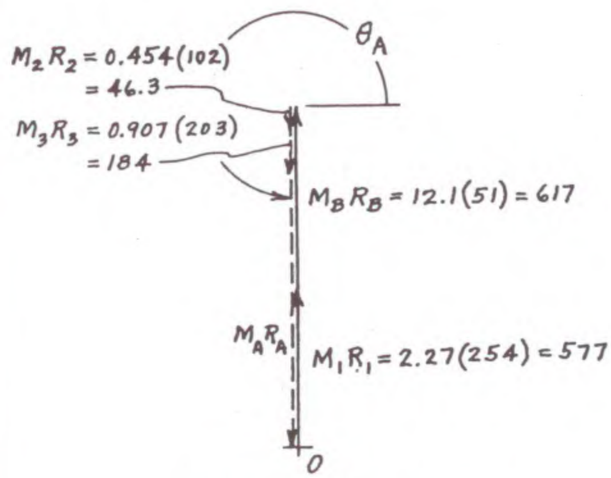
Moment Balance

Scale: 1mm = 1200 MRa units

$M_B R_B a_B$ scales 62800 units

$$M_B = \frac{62800}{R_B a_B} = \frac{62800}{51(102)} = \underline{\underline{12.1 \text{ kg}}}$$

$\underline{\underline{\theta_B = 90°}}$



Force Balance

Scale: 1mm = 20 MR units

$M_A R_A$ scales 954 units

$$M_A = \frac{954}{R_A} = \frac{954}{51} = \underline{\underline{18.7 \text{ kg}}}$$

$\underline{\underline{\theta_A = 270°}}$

19-3

No.	M, kg	R, mm	θ, deg	a, mm	cos θ	sin θ	MR cos θ	MR sin θ	MRa cos θ	MRa sin θ
1	2.27	254	90	-76	0	1	0	577	0	-43900
2	0.454	102	270	0	0	-1	0	-46.3	0	0
3	0.907	203	270	102	0	-1	0	-184	0	-18800
							Σ = 0	Σ = 346.7	Σ = 0	Σ = -62 00

Moment balance:

$$\Sigma MRa \sin\theta + M_B R_B a_B \sin\theta_B = 0$$

$$-62700 + M_B R_B a_B \sin\theta_B = 0 \qquad (1)$$

$$\Sigma MRa \cos\theta + M_B R_B a_B \cos\theta_B = 0$$

$$0 + M_B R_B a_B \cos\theta_B = 0 \qquad (2)$$

From Eqs. (1) and (2),

$$\frac{M_B R_B a_B \sin\theta_B}{M_B R_B a_B \cos\theta_B} = \frac{62700}{0} \text{ or } \tan\theta_B = \infty$$

Because $\sin\theta_B$ is (+) and $\cos\theta_B$ is (0), $\underline{\underline{\theta_B = 90^\circ}}$

From Eq. (1),

$$M_B R_B a_B \sin\theta_B = 62700$$

$$M_B(51)102(1) = 62700, \quad M_B = \frac{62700}{5202} = \underline{\underline{12.1 \text{ kg}}}$$

Force balance:

$$\Sigma MR\cos\theta + M_B R_B \cos\theta_B + M_A R_A \cos\theta_A = 0$$

$$0 + 12.1(51)0 + M_A R_A \cos\theta_A = 0 \qquad (3)$$

$$\Sigma MR\sin\theta + M_B R_B \sin\theta_B + M_A R_A \sin\theta_A = 0$$

$$347 + 12.1(51)1 + M_A R_A \sin\theta_A = 0 \qquad (4)$$

From Eqs. (3) and (4),

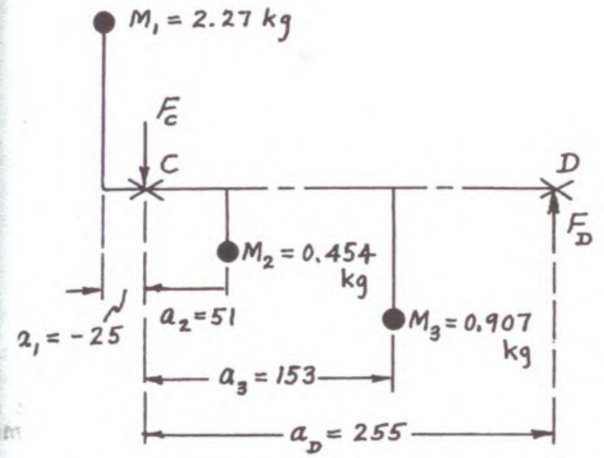
$$\frac{M_A R_A \sin\theta_A}{M_A R_A \cos\theta_A} = \frac{-964}{0} \text{ or } \tan\theta_A = -\infty$$

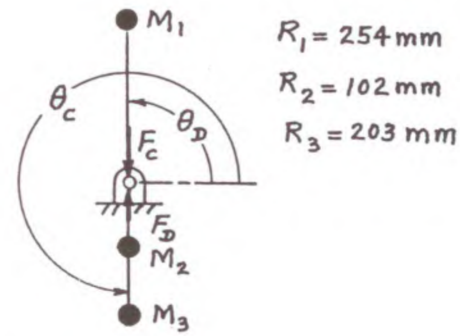
Because $\sin\theta_A$ is (−) and $\cos\theta_A$ is (0), $\underline{\underline{\theta_A = 270^\circ}}$

From Eq. (4), $M_A R_A \sin\theta_A = -964$

$$M_A(51)(-1) = -964, \quad M_A = \frac{964}{51} = \underline{18.9 \text{ kg}}$$

19-4

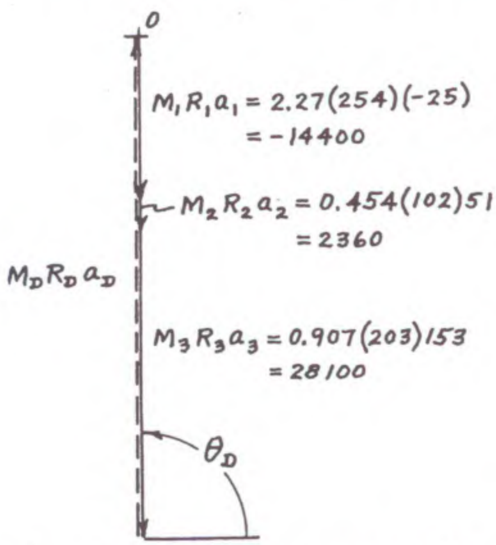




$$F = MR\omega^2$$

$$= M\frac{R}{1000}\left(\frac{2\pi \times 1000}{60}\right)^2 = 11.0\,MR$$

Balance of moments about a plane through C:
Let F_c and F_D be bearing reactions.



Scale: 1 mm = 600 MRa units

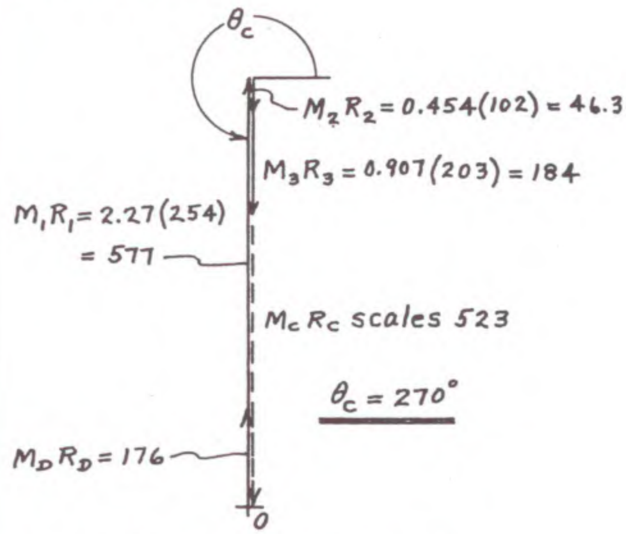
$M_DR_Da_D$ scales 44860 units

$$M_DR_D = \frac{44860}{a_D} = \frac{44860}{255} = 176$$

$$F_D = 11.0\,M_DR_D = 11.0\,(176)$$
$$= \underline{\underline{1936\text{ N}}}$$

$\underline{\underline{\theta_D = 90^\circ}}$

Force balance:



Scale: 1 mm = 10 MR units

$$F_c = 11.0\,M_cR_c = 11.0(523)$$
$$= \underline{\underline{5750\text{ N}}}$$

19-5

See figure in solution to Prob. 19-4.

Moment balance about a plane through C:

No.	M, kg	R, mm	θ, deg	a, mm	cos θ	sin θ	MR cos θ	MR sin θ	MRa cos θ	WRa sin θ
1	2.27	254	90	−25	0	1	0	577	0	−14425
2	0.454	102	270	51	0	−1	0	−46.3	0	−2361
3	0.907	203	270	153	0	−1	0	−184	0	−28152
							Σ = 0	Σ = 347	Σ = 0	Σ = −44938

$$\Sigma MRa\sin\theta + M_D R_D a_D \sin\theta_D = 0$$

$$-44938 + M_D R_D a_D \sin\theta_D = 0 \qquad (1)$$

$$\Sigma MRa\cos\theta + M_D R_D a_D \cos\theta_D = 0$$

$$0 + M_D R_D a_D \cos\theta_D = 0 \qquad (2)$$

$$F = MR\omega^2$$

$$= M\frac{R}{1000}\left(\frac{2\pi \times 1000}{60}\right)^2 \quad \text{where } R \text{ is in mm}$$

$$= 11.0\,MR$$

From Eqs. (1) and (2), $\dfrac{\sin\theta_D}{\cos\theta_D} = \dfrac{44938}{0}$ or $\tan\theta_D = \infty$

Because $\sin\theta_D$ is (+) and $\cos\theta_D$ is (0), $\underline{\underline{\theta_D = 90^\circ}}$

From Eq. (1), $M_D R_D (255) 1 = 44938$ or $M_D R_D = 176$, $F_D = 11.0\, M_D R_D$

$= 11.0(176) = \underline{\underline{1936\ \text{N}}}$

Force balance:

$$\Sigma MR\cos\theta + M_D R_D \cos\theta_D + M_C R_C \cos\theta_C = 0$$

$$0 + 17.6(0) + M_C R_C \cos\theta_C = 0 \qquad (3)$$

$$\Sigma MR\sin\theta + M_D R_D \sin\theta_D + M_C R_C \sin\theta_C = 0$$

$$347 + 176(1) + M_C R_C \sin\theta_C = 0 \qquad (4)$$

From Eqs. (3) and (4), $\dfrac{\sin\theta_C}{\cos\theta_C} = \dfrac{-523}{0}$ or $\tan\theta_C = -\infty$

Because $\sin\theta_C$ is (−) and $\cos\theta_C$ is (0), $\underline{\underline{\theta_C = 270^\circ}}$

From Eq. (4), $M_C R_C \sin\theta_C = -523$

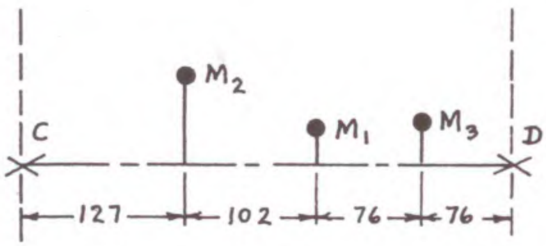
$$M_C R_C(-1) = -523, \quad M_C R_C = 523$$

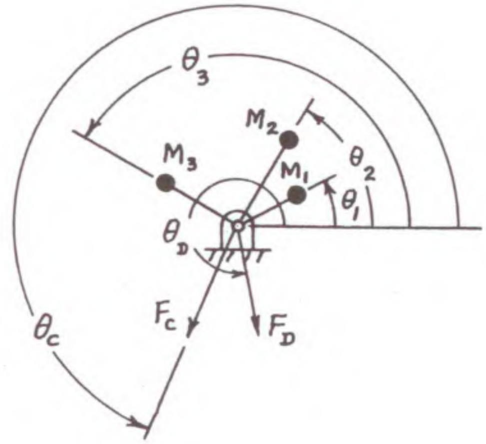
$F_C = 11.0\, M_C R_C = 11.0(523) = \underline{\underline{5750\ \text{N}}}$

19-

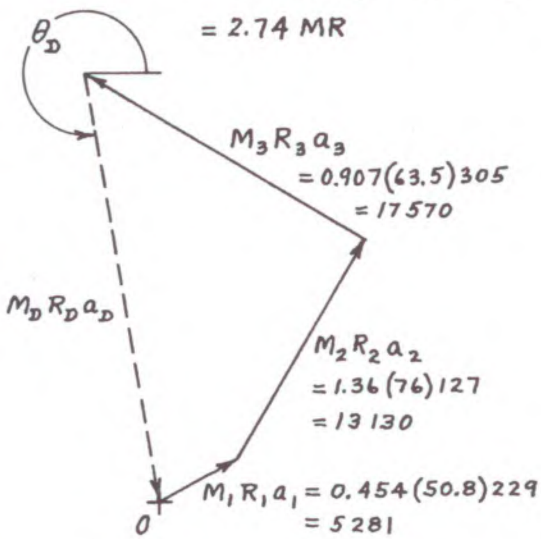
Mor

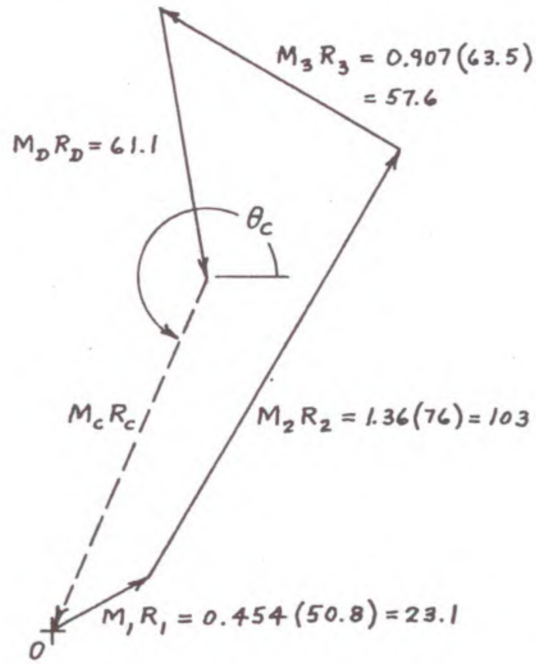
19-6





$$F = MR\omega^2 = M\frac{R}{1000}\left(\frac{2\pi \times 500}{60}\right)^2$$
$$= 2.74\ MR$$





Moment Balance about Plane C

Scale: 1 mm = 350 MRa units

$M_D R_D a_D$ scales 23 300

$$M_D R_D = \frac{23\,300}{a_D} = \frac{23\,300}{381} = 61.1$$

$$F_D = 2.74\, M_D R_D = 2.74\,(61.1) = \underline{167\ N}$$

θ_D measures $280°$

Force Balance

Scale: 1 mm = 1.35 MR units

$M_c R_c$ scales 77.2

$$F_c = 2.74\, M_c R_c = 2.74\,(77.2) = \underline{212\ N}$$

θ_c measures $245°$

19-7

See figure in solution to Prob. 19-6.

$$F = MR\omega^2 = M\frac{R}{1000}\left(\frac{2\pi \times 500}{60}\right)^2 = 2.74\,MR$$

Balance of moments about plane C:

No.	M, kg	R, mm	θ, deg.	a, mm	cos θ	sin θ	MR cos θ	MR sin θ	MRa cos θ	MRa sin θ
1	0.454	50.8	30	229	0.866	0.500	19.97	11.53	4573	2640
2	1.36	76.0	60	127	0.500	0.866	51.68	89.51	6563	11368
3	0.907	63.5	150	305	−0.866	0.500	−49.87	28.80	−15210	8784
							Σ = 21.78	Σ = 129.84	Σ = −4074	Σ = 22792

$$\Sigma MRa \sin\theta + M_D R_D a_D \sin\theta_D = 0$$
$$22792 + M_D R_D a_D \sin\theta_D = 0 \qquad (1)$$

$$\Sigma MRa \cos\theta + M_D R_D a_D \cos\theta_D = 0$$
$$-4074 + M_D R_D a_D \cos\theta_D = 0 \qquad (2)$$

From Eqs. (1) and (2), $\dfrac{\sin\theta_D}{\cos\theta_D} = \dfrac{-22792}{4074}$ or $\tan\theta_D = -5.595$

Because $\sin\theta_D$ is (−) and $\cos\theta_D$ is (+), θ_D lies in 4th quadrant.

$\theta_D = \tan^{-1}(-5.595) = \underline{280.1^\circ}$

From Eq. (1), $M_D R_D a_D \sin\theta_D = -22792$, $M_D R_D (381)(-0.9845) = -22792$

$M_D R_D = \dfrac{22792}{381(0.9845)} = 60.76$, $F_D = 2.74 M_D R_D = 2.74(60.76) = \underline{\underline{166\ N}}$

Force balance:

$$\Sigma MR\cos\theta + M_D R_D \cos\theta_D + M_C R_C \cos\theta_C = 0$$
$$21.78 + 60.76(0.1754) + M_C R_C \cos\theta_C = 0 \qquad (3)$$

$$\Sigma MR\sin\theta + M_D R_D \sin\theta_D + M_C R_C \sin\theta_C = 0$$
$$129.84 + 60.76(-0.9845) + M_C R_C \sin\theta_C = 0 \qquad (4)$$

From Eqs. (3) and (4), $\dfrac{\sin\theta_C}{\cos\theta_C} = \dfrac{-70.02}{-32.44}$ or $\tan\theta_C = 2.16$

Because $\sin\theta_C$ is (−) and $\cos\theta_C$ is (−), θ_C lies in 3rd quadrant.

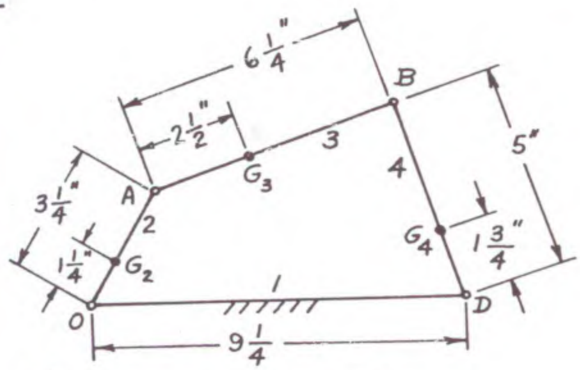
$\underline{\underline{\theta_C = 245.1^\circ}}$

From Eq. (4), $129.84 + 60.76(-0.9845) + M_C R_C(-0.9070) = 0$

Then $70.02 = M_C R_C(0.9070)$, $M_C R_C = 77.20$

$F_C = 2.74\,M_C R_C = 2.74(77.20) = \underline{\underline{212\ N}}$

20-1

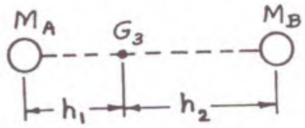


$M_2 = 1\frac{1}{2}$ lb

$M_3 = 3$ lb

$M_4 = 2\frac{1}{4}$ lb

a)



$$M_A + M_B = 3 \qquad (1)$$

$$M_A h_1 = M_B h_2 \qquad (2)$$

$$2.5\,M_A = 3.75\,M_B$$

or

$$M_B = \frac{2.5}{3.75} M_A = 0.667\,M_A \qquad (3)$$

Substit. Eq. (3) into Eq. (1),

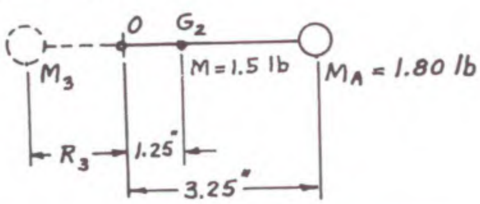
$$M_A + 0.667\,M_A = 3$$

$$M_A = \frac{3}{1.667} = \underline{\underline{1.80 \text{ lb}}}$$

$$M_B = 3 - M_A$$

$$= 3 - 1.80 = \underline{\underline{1.20 \text{ lb}}}$$

b) For link 2:

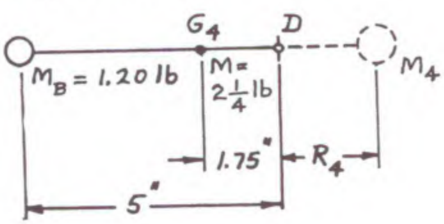


Σ moments about point O = zero :

$$M_3 R_3 = 1.5(1.25) + 1.80(3.25)$$

$$= 1.875 + 5.85 = \underline{7.72 \text{ in-lb}}$$

For link 4:



Σ moments about point D = zero:

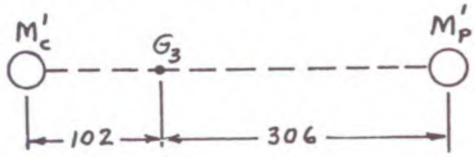
$$M_4 R = 2.25(1.75) + 1.20(5)$$

$$= 3.94 + 6 = \underline{9.94 \text{ in-lb}}$$

20-2

a) $\omega = \dfrac{1500(2\pi)}{60} = 157 \text{ rad/s}$

$M_c'' = \dfrac{R_2}{R} M_c = \dfrac{63.5}{102}(3.63) = 2.26 \text{ kg}$



$M_c' + M_p' = 3.63 \qquad (1)$

$M_c'(102) = M_p'(306), \quad M_c' = M_p'\left(\dfrac{306}{102}\right)$

$M_c' = 3 M_p' \qquad (2)$

Substitute Eq. (2) into Eq. (1).

$3M_p' + M_p' = 3.63, \quad 4M_p' = 3.63$

$M_p' = 0.908 \text{ kg}$

$M_c' = 3.63 - M_p' = 3.63 - 0.908 = 2.722 \text{ kg}$

Total mass at crankpin

$= M_c'' + M_c' = 2.26 + 2.72 = 4.98 \text{ kg}$

Total mass at piston pin

$= M_p + M_p' = 3.18 + 0.908 = 4.09 \text{ kg}$

$f_c = (M_c'' + M_c')R\omega^2 = 4.98(0.102)(157)^2$

$= 12\,500 \text{ N}$

Primary inertia force

$= (M_p + M_p')R\omega^2 \cos\theta$

$= 4.09(0.102)(157)^2 \cos\theta$

$= 10\,300 \cos\theta \text{ N}$

Secondary inertia force

$= (M_p + M_p')R\omega^2 \dfrac{R}{L} \cos 2\theta$

$= 4.09(0.102)(157)^2 \dfrac{102}{408} \cos 2\theta$

$= 2570 \cos 2\theta \text{ N}$

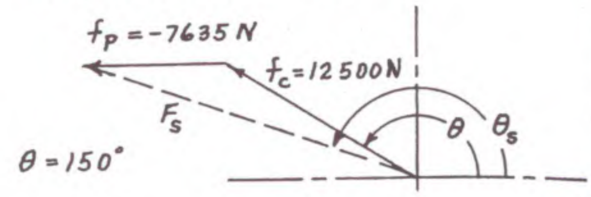
Total inertia force

$f_p = 10\,300 \cos\theta + 2570 \cos 2\theta$

For $\theta = 150°$,

$f_p = 10300(-0.866) + 2570(0.500$

$= -8920 + 1285 = -7635 \text{ N}$



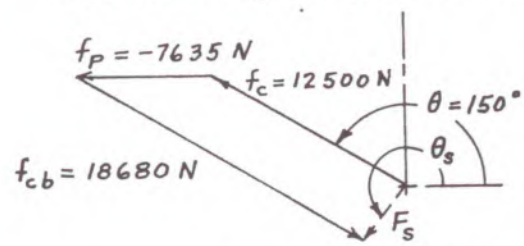
Scale: 1mm = 350 N

$\underline{F_s \text{ scales } 19700 \text{ N}}$

$\underline{\theta_s \text{ measures } 161.5°}$

b) $f_{cb} = f_c + 0.6(\text{max. primary inertia force})$

$= 12\,500 + 0.6(10\,300) = 18\,680 \text{ N}$



Scale: 1mm = 350 N

$\underline{F_s \text{ scales } 3850 \text{ N}}$

$\underline{\theta_s \text{ measures } 234°}$

c) $f_{cb} = M_{cb} r\omega^2$

or $M_{cb} = \dfrac{f_{cb}}{r\omega^2} = \dfrac{18680}{0.0508(157)^2}$

$= \underline{14.9 \text{ kg}}$

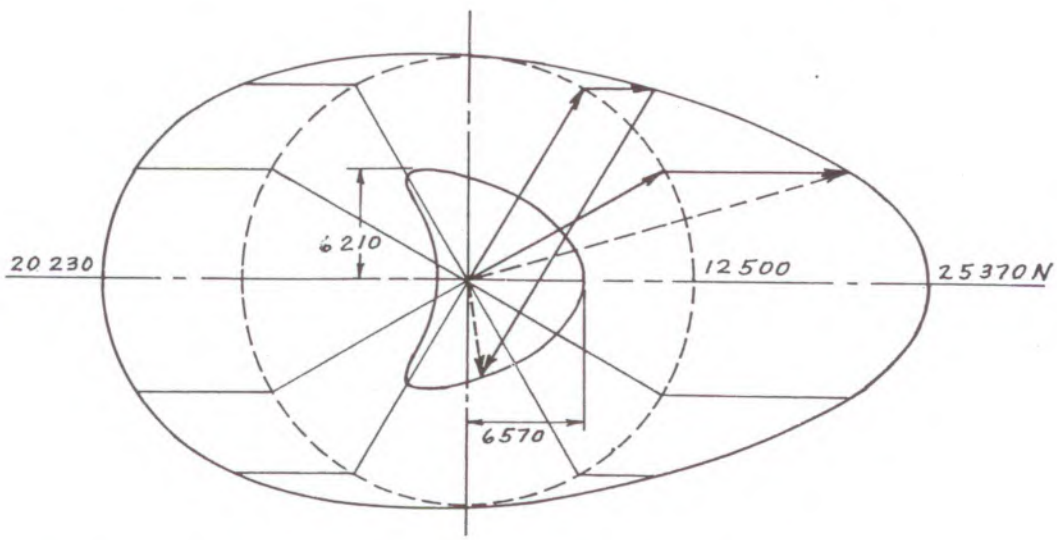
20-3

a) From solution to Prob. 20-2, $f_c = 12500\,N$

$$f_p = \underbrace{10\,300 \cos\theta}_{\text{Primary}} + \underbrace{2570 \cos 2\theta}_{\text{Secondary}}$$

$$f_{cb} = 12\,500 + 0.6\,(10\,300) = 18\,680\ N$$

θ	$\cos\theta$	2θ	$\cos 2\theta$	Primary f_P, N	Secondary f_P, N	Resultant f_P, N
0	1.000	0	1.000	10 300	570	12 870
30	0.866	60	0.500	8 920	1 285	10 205
60	0.500	120	−0.500	5 150	−1 285	3 865
90	0	180	−1.000	0	−2 570	−2 570
120	−0.500	240	−0.500	−5 150	−1 285	−6 435
150	−0.866	300	0.500	−892	1285	−7635
180	−1.000	360	1.000	−10 300	2 570	−7730
210	−0.866	420	0.500	−8 920	1 285	−7635
240	−0.500	480	−0.500	−5 150	−1 285	−6435
270	0	540	−1.000	0	−2 570	−2570
300	0.500	600	−0.500	5150	−1 285	3865
330	0.866	660	0.500	8 920	1285	10 205
360	1.000	720	1.000	10 300	2570	12 870

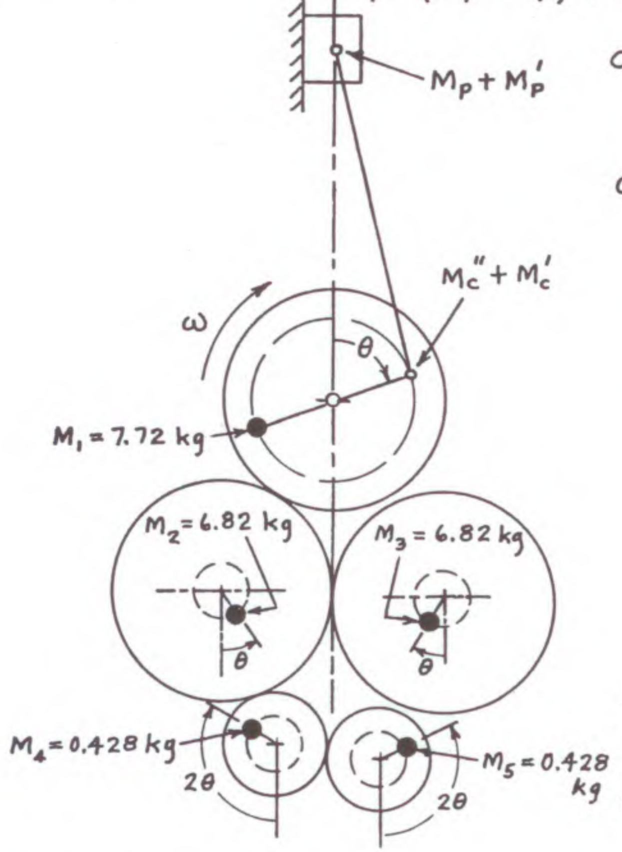


b) Max. horizontal shaking force scales $\underline{\underline{6570\ N}}$

" vertical " " " $\underline{\underline{6210\ N}}$

20-4

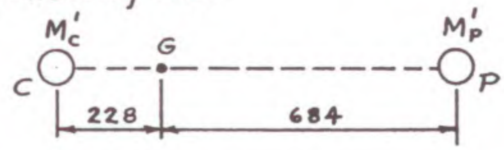
$$f_p = (M_p + M_p')R\omega^2\cos\theta + (M_p + M_p')R\omega^2\frac{R}{L}\cos 2\theta$$



Crank:

$$M_c'' = \frac{R_2}{R} M_c = \frac{121}{229}(4.30) = 2.27 \text{ kg}$$

Connecting rod:



$$M_c' + M_p' = 7.26 \qquad (1)$$

$$M_c'(228) = M_p'(684), \quad M_c' = \frac{684}{228} M_p'$$

$$M_c' = 3 M_p' \qquad (2)$$

Substitute Eq. (2) into Eq. (1).

$$3M_p' + M_p' = 7.26, \quad 4M_p' = 7.26$$

$$M_p' = 1.815 \text{ kg}$$

$$M_c' = 7.26 - M_p' = 7.26 - 1.815 = 5.45 \text{ kg}$$

Total mass at crankpin

$$= M_c'' + M_c' = 2.27 + 5.45 = 7.72 \text{ kg}$$

Total mass at piston pin

$$= M_p + M_p' = 2.72 + 1.815 = 4.54 \text{ kg}$$

Want M_1 to balance $M_c'' + M_c'$

$$M_1(229) = (M_c'' + M_c')229$$

$$M_1(229) = (7.72)229, \quad \underline{\underline{M_1 = 7.72 \text{ kg}}}$$

Primary inertia force

$$= (M_p + M_p')R\omega^2\cos\theta$$

$$= 4.54(0.229)\omega^2\cos\theta$$

When $\theta = 0°$

$$(M_2 + M_3)(0.0762)\omega^2 = 4.54(0.229)\omega^2$$

$$M_2 + M_3 = 4.54\left(\frac{0.229}{0.0762}\right) = 13.64 \text{ kg}$$

$$\underline{\underline{M_2 = M_3 = \frac{13.64}{2} = 6.82 \text{ kg}}}$$

Secondary inertia force

$$= (M_p + M_p')R\omega^2\frac{R}{L}\cos 2\theta$$

$$= 4.54(0.229)\omega^2\frac{229}{912}\cos 2\theta$$

$$= (0.261)\omega^2\cos 2\theta$$

When $\theta = 0°$

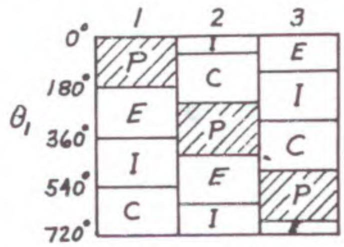
$$(M_4 + M_5)(0.0762)(2\omega)^2 = 0.261\omega^2$$

$$M_4 + M_5 = \frac{0.261}{0.0762} = 0.856 \text{ kg}$$

$$\underline{\underline{M_4 = M_5 = \frac{0.856}{2} = 0.428 \text{ kg}}}$$

20-5

a)

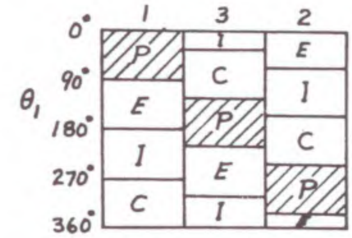


$\frac{720}{3} = 240$

Four - stroke cycle

Power strokes occur at 240° intervals

b)



$\frac{360}{3} = 120$

Two - stroke cycle

Power strokes occur at 120° intervals

20-6

a)

Crank	ϕ	$\cos\phi$	$\sin\phi$	2ϕ	$\cos 2\phi$	$\sin 2\phi$	a	$a\cos\phi$	$a\sin\phi$	$a\cos 2\phi$	$a\sin 2\phi$
1	0°	1.000	0	0	1.000	0	0	0	0	0	0
2	120°	-0.500	0.866	240	-0.500	-0.866	a	-0.500 a	0.866 a	-0.500 a	-0.866 a
3	240°	-0.500	-0.866	480	-0.500	0.866	2a	-a	-1.732 a	-a	1.732 a
	Σ	0	0		0	0		-1.500 a	-0.866 a	-1.500 a	0.866 a
		Primary forces balanced			Secondary forces balanced			Primary moments unbalanced		Secondary moments unbalanced	

b) $\underline{F_s = 0}$

c) $$M = (M_p + M_p')R\omega^2 \Big[-1.500\,a\cos\theta_1 + 0.866\,a\sin\theta_1 - \frac{R}{L}(1.500\,a\cos 2\theta_1) - \frac{R}{L}(0.866\,a\sin 2\theta_1) \Big]$$

d) M is a pure couple because $F_s = 0$

20-7

a)

Crank	ϕ	$\cos\phi$	$\sin\phi$	2ϕ	$\cos 2\phi$	$\sin 2\phi$	a	$a\cos\phi$	$a\sin\phi$	$a\cos 2\phi$	$a\sin 2\phi$
1	0°	1	0	0	1	0	0	0	0	0	0
2	180°	-1	0	360	1	0	a	-a	0	a	0
3	0°	1	0	0	1	0	2a	2a	0	2a	0
4	180°	-1	0	360	1	0	3a	-3a	0	3a	0
	Σ	0	0		4	0		-2a	0	6a	0
		Primary forces balanced			Secondary forces unbalanced			Primary moments unbalanced		Secondary moments unbalanced	

b) $$F_s = (M_p + M_p')R\omega^2 \left[\frac{R}{L}(4)\cos 2\theta_1 \right]$$

c) $$M = (M_p + M_p')R\omega^2 \left[-2a\cos\theta_1 + \frac{R}{L}\,6a\cos 2\theta_1 \right]$$

20-7 (CONT.)

d) $$z = \frac{M}{F_s} = \frac{-2a\cos\theta_1 + 6\frac{Ra}{L}\cos 2\theta_1}{4\frac{R}{L}\cos 2\theta_1}$$

$$= \frac{-a\cos\theta_1 + 3\frac{Ra}{L}\cos 2\theta_1}{2\frac{R}{L}\cos 2\theta_1}$$

20-8

a) $\frac{720}{4} = 180$ $\therefore$ for firing at equal time intervals, firing must occur after each 180° of rotation.

Firing orders possible if used as four-stroke cycle:

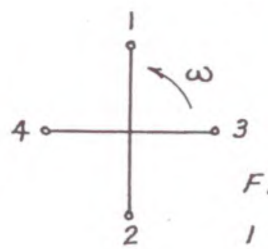
1 2 3 4

1 4 3 2

b) Firing order if used as a two-stroke cycle:

1 and 3, then 2 and 4

c) $\frac{360}{4} = 90°$ $\therefore$ for firing at equal time intervals, firing must occur after each 90° of rotation.



Firing order:

1 3 2 4

20-9

Crank	ϕ	$\cos\phi$	$\sin\phi$	2ϕ	$\cos 2\phi$	$\sin 2\phi$	a	$a\cos\phi$	$a\sin\phi$	$a\cos 2\phi$	$a\sin 2\phi$
1	0°	1	0	0	1	0	0	0	0	0	0
2	240°	-0.5	-0.866	480	-0.5	0.866	a	-0.5a	-0.866a	-0.5a	0.866a
3	120°	-0.5	0.866	240	-0.5	-0.866	2a	-a	1.732a	-a	-1.732a
4	120°	-0.5	0.866	240	-0.5	-0.866	3a	-1.5a	2.598a	-1.5a	-2.598a
5	240°	-0.5	-0.866	480	-0.5	0.866	4a	-2a	-3.464a	-2a	3.464a
6	0°	1	0	0	1	0	5a	5a	0	5a	0
	Σ	0	0		0	0		0	0	0	0
		Primary forces balanced			Secondary forces balanced			Primary moments balanced		Secondary moments balanced	

20-10

$\frac{720}{6} = 120$ $\therefore$ for firing at equal time intervals, firing must occur after each 90° of rotation.

1 5 3 6 2 4 ← Given

1 5 4 6 2 3

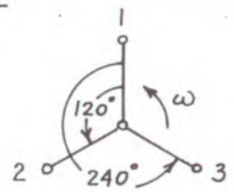
1 2 3 6 5 4

1 2 4 6 5 3

20-11

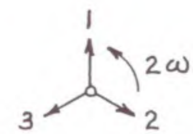
Crank	ϕ	$\cos\phi$	$\sin\phi$	2ϕ	$\cos 2\phi$	$\sin 2\phi$	a	$a\cos\phi$	$a\sin\phi$	$a\cos 2\phi$	$a\sin 2\phi$
1	$0°$	1	0	$0°$	1	0	0	0	0	0	0
2	$180°$	−1	0	$360°$	1	0	a	$-a$	0	a	0
3	$90°$	0	1	$180°$	−1	0	$2a$	0	$2a$	$-2a$	0
4	$270°$	0	−1	$540°$	−1	0	$3a$	0	$-3a$	$-3a$	0
5	$270°$	0	−1	$540°$	−1	0	$4a$	0	$-4a$	$-4a$	0
6	$90°$	0	1	$180°$	−1	0	$5a$	0	$5a$	$-5a$	0
7	$180°$	−1	0	$360°$	1	0	$6a$	$-6a$	0	$6a$	0
8	$0°$	1	0	$0°$	1	0	$7a$	$7a$	0	$7a$	0
	Σ	0	0		0	0		0	0	0	0
		Primary forces balanced			Secondary forces balanced			Primary moments balanced		Secondary moments balanced	

20-12

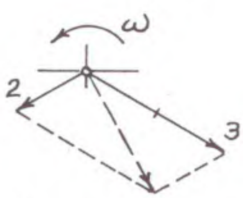


Cranks

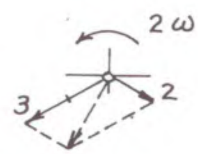
Primary force vectors balanced



Secondary force vectors balanced

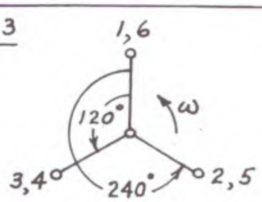


Primary moment vectors unbalanced



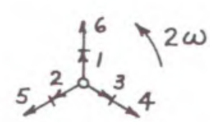
Secondary moment vectors unbalanced

20-13

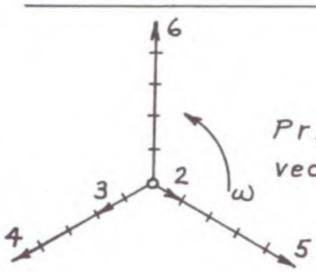


Cranks

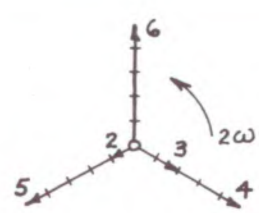
Primary force vectors balanced



Secondary force vectors balanced



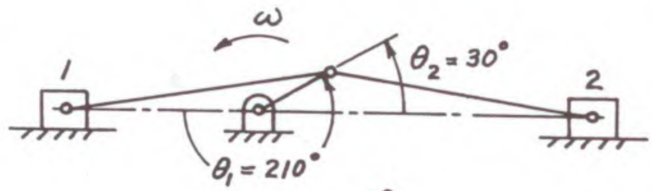
Primary moment vectors balanced

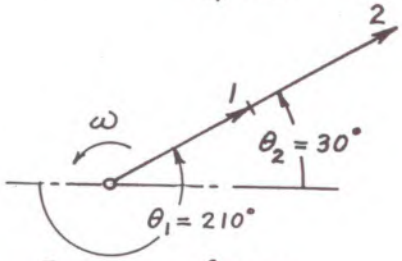


Secondary moment vectors balanced

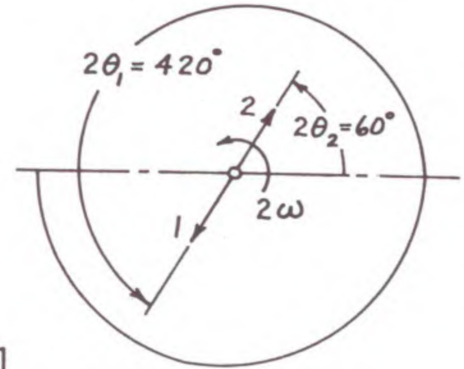
20-14

a)





Primary force vectors unbalanced



Secondary force vectors balanced

b)

$$\omega = \frac{4000\,(2\pi)}{60} = 419 \text{ rad/s}$$

$$(M_p + M_{p'})R\omega^2 = \frac{3.5}{386}(2)\,419^2$$

$$= \frac{7\,(175\,000)}{386} = 3180 \text{ lb}$$

$$f_p = (M_p + M_p')R\omega^2 \cos\theta + (M_p + M_p')R\omega^2 \frac{R}{L}\cos 2\theta$$

Piston No. 1: $f_{P_1} = 3180 \underbrace{\cos 210°}_{-0.866} + 3180\left(\frac{2}{6.5}\right)\underbrace{\cos 420°}_{0.500}$

$= -2750 + 489 = -2261$ lb $\quad \therefore f_{P_1} = \underline{2261 \text{ lb}} \rightarrow$

Piston No. 2: $f_{P_2} = 3180 \underbrace{\cos 30°}_{0.866} + 3180\left(\frac{2}{6.5}\right)\underbrace{\cos 60°}_{0.500}$

$= 2750 + 489 = 3239$ lb $\quad \therefore f_{P_2} = \underline{3239 \text{ lb}} \rightarrow$

Resultant $f_P = f_{P_1} + f_{P_2} = \underrightarrow{2261} + \underrightarrow{3239} = \underline{\underrightarrow{5500} \text{ lb}}$

c) Primary inertia force (max.) for 1 piston = 3180 lb

$$f_{cb} = 2(3180) = 6360 \text{ lb}$$

$$\frac{M_{cb}}{386} r\omega^2 = 6360$$

$$\frac{M_{cb}}{386}(1.5)\,419^2 = 6360, \quad M_{cb} = \frac{6360 \times 386}{1.5 \times 175\,000} = \underline{9.34 \text{ lb}}$$

20-15

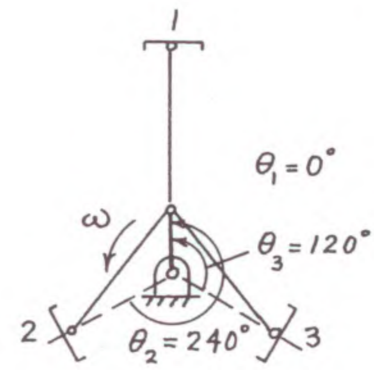
a) Concentrated mass at crank pin

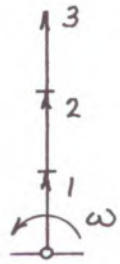
$= M_c'' + 3M_c'$

$M_{cb} r\omega^2 = (M_c'' + 3M_c') R\omega^2$

$$M_{cb} = \frac{R}{r}\left(M_c'' + 3M_c'\right)$$

b)



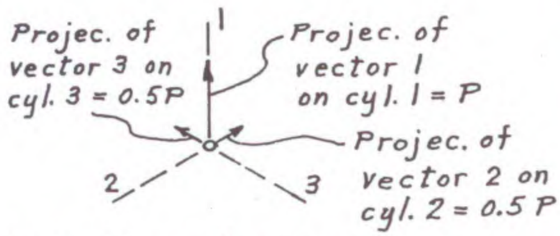


Magnitude of each = P

Primary force vectors

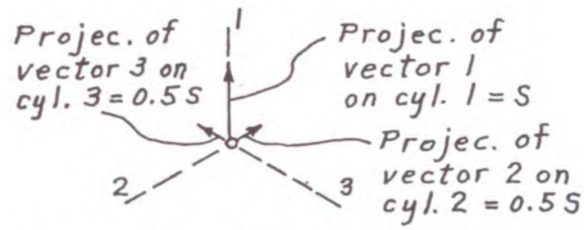
Magnitude of each = S

Secondary force vectors



Projections of primary force vectors unbalanced.

∴ Primary forces are unbalanced.



Projections of secondary force vectors unbalanced.

∴ Secondary forces are unbalanced.

c)

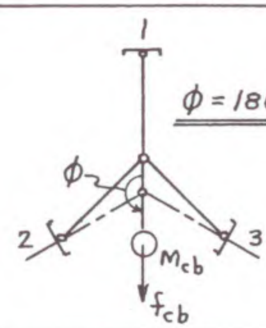
Resultant

$= P + 2(0.5P\cos 60°)$

$= 1.5P$

$= 1.5(M_p + M_p')R\omega^2$

d)



$$\phi = 180°$$

e) $M_{cb} r\omega^2 = 1.5(M_p + M_p')R\omega^2$

$$M_{cb} = \frac{1.5R}{r}\left(M_p + M_p'\right)$$

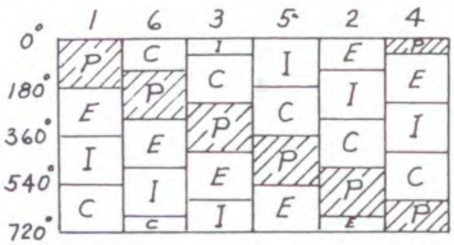
f) Total mass of counterbalance

$= \frac{R}{r}\left(M_c'' + 3M_c'\right) + \frac{1.5R}{r}\left(M_p + M_p'\right)$

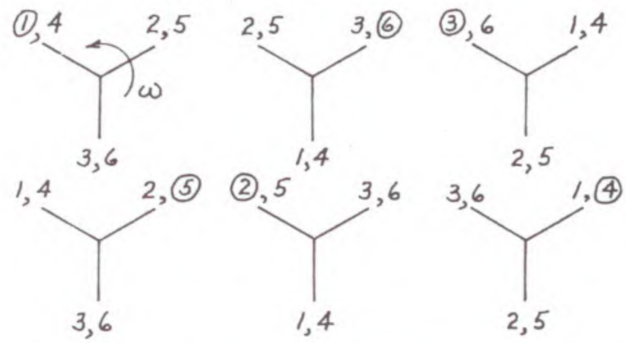
$$= \frac{R}{r}\left[M_c'' + 3M_c' + 1.5\left(M_p + M_p'\right)\right]$$

20-16

a)

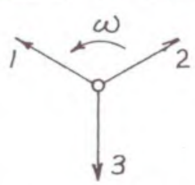


Successive crank positions:
Firing cyl. encircled.



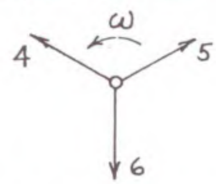
b)

Left Bank

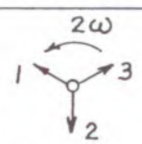


Primary force
vectors balanced

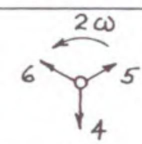
Right Bank



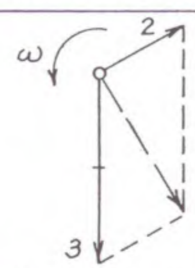
Primary force
vectors balanced



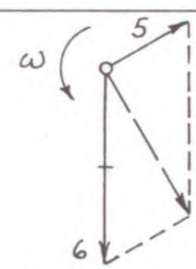
Secondary force
vectors balanced



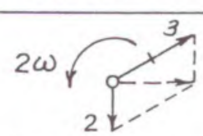
Secondary force
vectors balanced



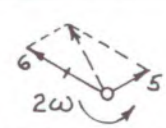
Primary moment
vectors unbalanced



Primary moment
vectors unbalanced



Secondary moment
vectors unbalanced



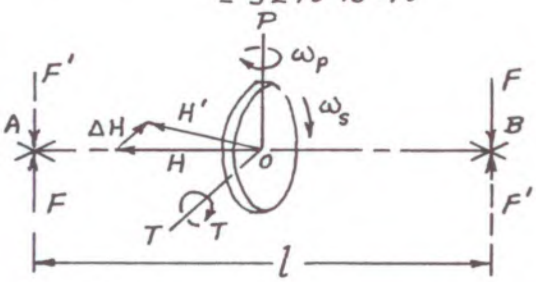
Secondary moment
vectors unbalanced

21-1

$$\omega_s = \frac{1800(2\pi)}{60} = 188 \text{ rad/s}$$

$$\omega_p = 0.5\left(\frac{\pi}{100}\right) = 0.00873 \text{ rad/s}$$

$$T = I\omega_s\omega_p = 3200(188)0.00873 = 5270 \text{ lb-ft}$$



Forces bearings exert on shaft are F.

$$F = \frac{T}{l} = \frac{5270}{15} = 351 \text{ lb}$$

Forces shaft exerts on bearings are $\underline{\underline{F' = 351 \text{ lb}}}$

21-2

a) $$\omega_s = \frac{10000(2\pi)}{60} = 1047 \text{ rad/s}$$

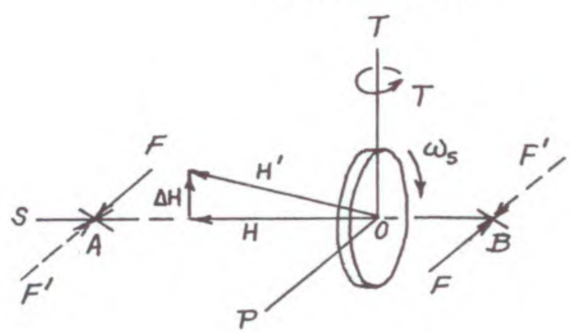
Plane velocity,

$$V = \frac{965000}{60 \times 60} = 268 \text{ m/s}$$

$$\omega_P = \frac{V}{R} = \frac{268}{1524} = 0.176 \text{ rad/s}$$

$$I = Mk^2 = 816(0.229)^2 = 42.8 \text{ kg}\cdot\text{m}^2$$

$$T = I\omega_s\omega_P = 42.8(1047)0.176 = 7890 \text{ N}\cdot\text{m}$$

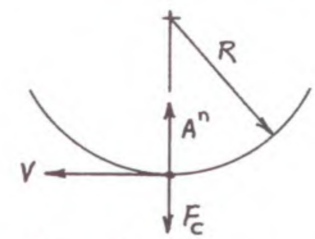


21-2 (CONT.)

$$Fl = T, \quad F = \frac{T}{l} = \frac{7890}{2.286} = 3450 \text{ N}$$

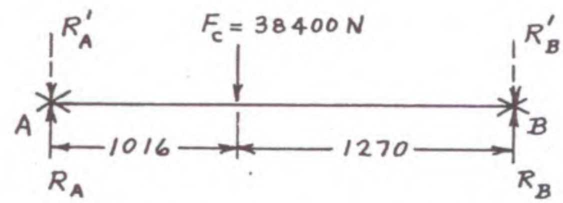
Forces brgs. exert on shaft are F.
Forces shaft exerts on brgs. are $\underline{\underline{F' = 3450 \text{ N}}}$

b)



$$A^n = \frac{V^2}{R} = \frac{(268)^2}{1524} = 47.1 \text{ m/s}^2$$

$$F_c = MA^n = 816(47.1) = 38400 \text{ N}$$



Forces brgs. exert on shaft are R_A and R_B.

$\Sigma M_A = 0$: $\quad 2286 R_B - 38400(1016) = 0$

$$R_B = 38400\left(\frac{1016}{2286}\right) = 17067 \text{ N}$$

$\Sigma F_y = 0$: $\quad R_A = 38400 - 17067 = 21333 \text{ N}$

Forces shaft exerts on brgs. are R'_A and R'_B.

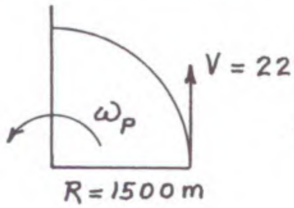
$\underline{\underline{R'_A = 21333 \text{ N}}} \qquad \underline{\underline{R'_B = 17067 \text{ N}}}$

21-3

a) $\omega_s = \dfrac{8000(2\pi)}{60} = 838 \text{ rad/s}$

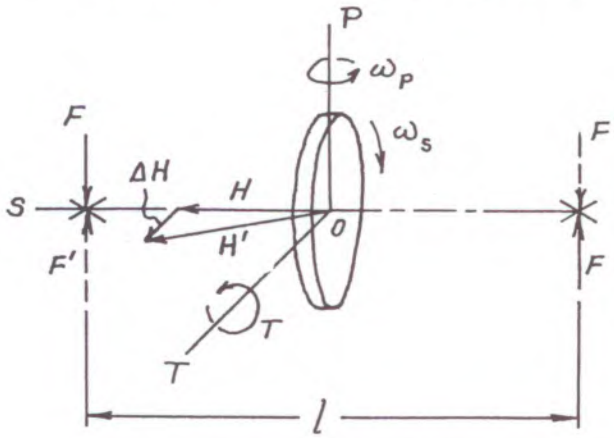
Plane velocity,

$$V = \frac{800\,000}{60 \times 60} = 222 \text{ m/s}$$



$V = 222$ m/s

$$\omega_p = \frac{V}{R} = \frac{222}{1500} = 0.148 \text{ rad/s}$$



$$I = Mk^2 = 816\,(0.229)^2 = 42.8 \text{ kg·m}^2$$

$$T = I\,\omega_s\,\omega_p = 42.8\,(838)\,0.148 = 5308 \text{ N·m}$$

$$T = Fl, \quad F = \frac{T}{l} = \frac{5308}{2.286} = 2\,322 \text{ N}$$

Forces brgs. exert on shaft are F.

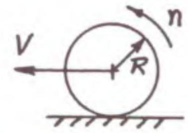
Forces shaft exerts on brgs. are $F' = 2322$ N

b) F' forces make front of plane tilt upward

21-4

V is car speed, m/s

n is angular speed of wheel, r/s



$$R = \frac{D}{2} = \frac{840}{2} = 420 \text{ mm}$$

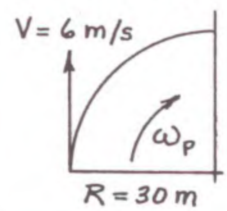
$$V = 2\pi R n$$

$$n = \frac{V}{2\pi R} = \frac{6}{2\pi\,(0.420)} = 2.27 \text{ r/s}$$

Armature speed

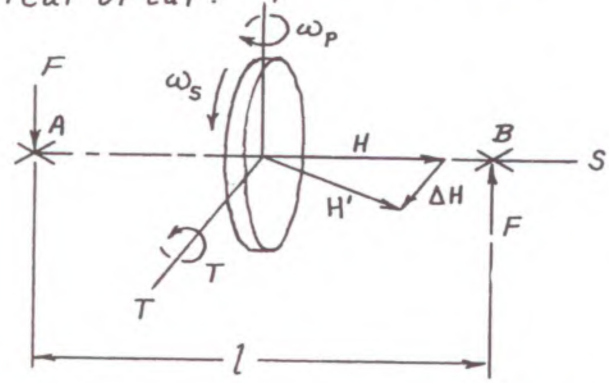
$$\omega_s = 4\,(2.27) = 9.08 \text{ r/s} = 9.08\,(2\pi) = 57.1 \text{ rad/s}$$

To find ω_p:



$$\omega_p = \frac{V}{R} = \frac{6}{30} = 0.2 \text{ rad/s}$$

View of armature shaft from rear of car:



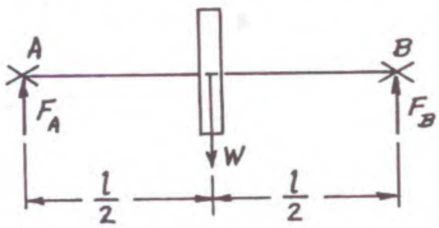
$$I = Mk^2 = 272\,(0.152)^2 = 6.284 \text{ kg·m}^2$$

$$T = I\,\omega_s\,\omega_p = 6.284\,(57.1)\,0.2 = 71.76 \text{ N·m}$$

$$T = Fl, \quad F = \frac{T}{l} = \frac{71.76}{0.610} = 118 \text{ N}$$

21-4 (CONT.)

Bearing reactions due to gravity force:



$$W = Mg = 272\,(9.806) = 2\,667 \text{ kg}\cdot\text{m/s}^2$$
$$= 2\,667 \text{ N}$$

$$F_A = F_B = \frac{2\,667}{2} = 1334 \text{ N}$$

Resultant forces that bearings exert on shaft are

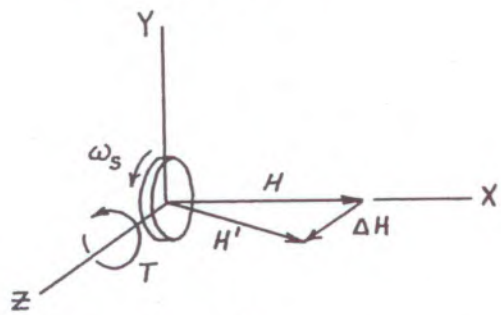
$$R_A = F_A - F = 1334 - 118 = \underline{\underline{1216 \text{ N up}}}$$

$$R_B = F_B + F = 1334 + 118 = \underline{\underline{1452 \text{ N up}}}$$

<u>21-5</u>

$$\omega_s = \frac{12\,000\,(2\pi)}{60} = 1257 \text{ rad/s}$$

$$\omega_p = \frac{1\,(\pi)}{180\,(60)60} = 4.85 \times 10^{-6} \text{ rad/s}$$



$$T = I\,\omega_s\,\omega_p$$
$$= 0.00271\,(1257)\,4.85 \times 10^{-6}$$
$$= 16.5 \times 10^{-6} \text{ N}\cdot\text{m}$$

T, friction torque, must be applied about Z axis, hence through bearings E and F.

22-1

$$I = \frac{\pi d^4}{64} = \frac{\pi (0.5)^4}{64} = 0.00307 \text{ in}^4$$

$$y_{st} = \frac{WL^3}{48EI} = \frac{25(12)^3}{48(30\times 10^6)0.00307}$$

$$= .00978 \text{ in}$$

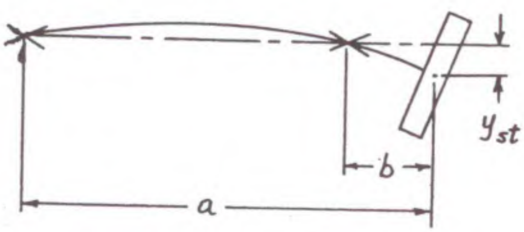
$$\omega_n = \sqrt{\frac{g}{y_{st}}} = \sqrt{\frac{386}{0.00978}} = 199 \text{ rad/s}$$

$$= \frac{199(60)}{2\pi} = \underline{\underline{1900 \text{ r/min}}}$$

Because $1200 < 0.8(1900)$, satisfactory

22-2

a) Simply supported:

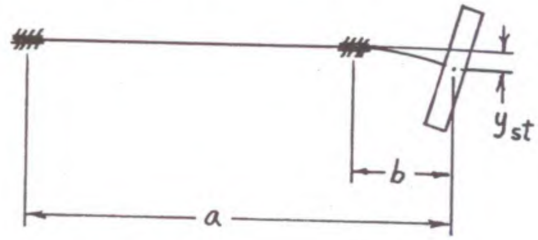


$$y_{st} = \frac{a b^2 Mg}{3EI}$$

$$\omega_n = \sqrt{\frac{g}{y_{st}}} = \underline{\underline{\sqrt{\frac{3EI}{ab^2M}}}}$$

22-2 (CONT)

b) Rigidly supported



$$y_{st} = \frac{Mgb^3}{3EI}$$

$$\omega_n = \sqrt{\frac{g}{y_{st}}} = \underline{\underline{\sqrt{\frac{3EI}{Mb^3}}}}$$

22-3

W, lb	y_{st}, in	y^2	Wy	Wy^2
60	0.0005	25×10^{-8}	0.0300	15.00×10^{-6}
30	0.0004	16×10^{-8}	0.0120	4.80×10^{-6}
20	0.0003	9×10^{-8}	0.0060	1.80×10^{-6}
		$\Sigma =$	0.0480	21.60×10^{-6}

$$\omega_n = \sqrt{\frac{g\Sigma Wy}{\Sigma Wy^2}} = \sqrt{\frac{386(0.0480)}{21.6\times 10^{-6}}}$$

$$= \sqrt{85.86\times 10^4} = 927 \text{ rad/s}$$

$$= 927\left(\frac{60}{2\pi}\right) = \underline{\underline{8852 \text{ r/min}}}$$

22-4

Cross sectional area:

$$A = \frac{\pi}{4}\left(D^2 - d^2\right) = \frac{\pi}{4}\left(0.063^2 - 0.057^2\right)$$

$$= \frac{\pi}{4}\left[\left(3.969\times10^{-3}\right) - \left(3.249\times10^{-3}\right)\right]$$

$$= 0.565\times10^{-3}\ \text{m}^2$$

M = mass of shaft

$$= AL \times \text{density}$$

$$= 0.565\times10^{-3}(1.422)\,7.83\times10^{3} = 6.29\ \text{kg}$$

$$I = \frac{\pi}{64}\left(D^4 - d^4\right) = \frac{\pi}{64}\left[\left(1.575\times10^{-5}\right) - \left(1.506\times10^{-5}\right)\right]$$

$$= \frac{\pi}{64}\left(0.519\times10^{-5}\right) = 2.55\times10^{-7}\ \text{m}^4$$

$$\omega_n = 9.87\sqrt{\frac{EI}{ML^3}}$$

22-4 (CONT.)

$$\omega_n = 9.87\sqrt{\frac{207\times10^{9}\left(2.55\times10^{-7}\right)}{6.29\,(2.88)}}$$

$$= 9.87\sqrt{2914} = 9.87\,(54)$$

$$= 532\ \text{rad/s}$$

$$= 532\,\frac{60}{2\pi} = \underline{\underline{5080\ \text{r/min}}}$$

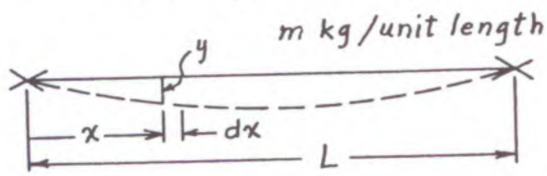
Max. operating speed is 4000 r/min.

$0.8\,(5080) = 4064$

Because $4000 < 4064$, satisfactory

22-5

$$\omega_n = \sqrt{\frac{g\,\Sigma My}{\Sigma My^2}} \qquad (1)$$



From a mechanical engineering handbook

$$y = \frac{mgx}{24EI}\left(L^3 - 2Lx^2 + x^3\right)$$

At any distance x in the figure we have a concentrated mass $= m\,dx$ and Eq. (1) becomes

$$\omega_n^2 = g\,\frac{\int_0^L m\,dx\,y}{\int_0^L m\,dx\,y^2} \qquad (2)$$

The numerator of Eq. (2) becomes

$$\frac{m^2 g}{24EI}\int_0^L\left(xL^3 - 2Lx^3 + x^4\right)dx = \frac{m^2 g}{24EI}\left[\frac{x^2L^3}{2} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]_0^L$$

$$= \frac{m^2 g}{24EI}\left(\frac{L^5}{2} - \frac{L^5}{2} + \frac{L^5}{5}\right) = \frac{m^2 g}{24EI}\,\frac{L^5}{5}$$

The denominator of Eq. (2) becomes

$$\frac{m^3 g^2}{(24)^2E^2I^2}\int_0^L x^2\left(L^3 - 2Lx^2 + x^3\right)^2 dx$$

$$= \frac{m^3 g^2}{(24)^2E^2I^2}\int_0^L\left(L^6 - 4L^4x^2 + 2L^3x^3 + 4L^2x^4 - 4Lx^5 + x^6\right)dx$$

22-5 (CONT.)

$$= \frac{m^3 g^2}{(24)^2 E^2 I^2} \int_0^L \left(x^2 L^6 - 4x^4 L^4 + 2x^5 L^3 + 4x^6 L^2 - 4x^7 L + x^8\right) dx$$

$$= \frac{m^3 g^2}{(24)^2 E^2 I^2} \left[\frac{x^3 L^6}{3} - \frac{4}{5} x^5 L^4 + 2\frac{x^6}{6} L^3 + 4\frac{x^7}{7} L^2 - 4\frac{x^8}{8} L + \frac{x^9}{9} \right]_0^L$$

$$= \frac{m^3 g^2}{(24)^2 E^2 I^2} \left(\frac{L^9}{3} - \frac{4}{5} L^9 + \frac{L^9}{3} + \frac{4}{7} L^9 - \frac{L^9}{2} + \frac{L^9}{9} \right) = \frac{m^3 g^2}{(24)^2 E^2 I^2} \frac{31}{630} L^9$$

Then

$$\omega_n^2 = g \frac{\frac{m^2 g}{24EI} \frac{L^5}{5}}{\frac{m^3 g^2}{(24)^2 E^2 I^2} \frac{31}{630} L^9} = \frac{EI\,(24)\,630}{m L^4\ (5)\,31} = 97.5 \frac{EI}{ML^3}$$

where $M = mL$

$$\omega_n = 9.87 \sqrt{\frac{EI}{ML^3}}$$

22-6

a) $I = \frac{\pi d^4}{64} = \frac{\pi}{64}(0.038)^4 = \frac{\pi}{64}(2.085 \times 10^{-6})$

$= 1.024 \times 10^{-7}\ m^4$

$A = \pi \frac{d^2}{4} = \frac{\pi(0.001444)}{4} = 1.134 \times 10^{-3}\ m^2$

$M = AL\ (\text{density}) = 1.134 \times 10^{-3}(0.914)\,7830$

$= 8.12\ kg$

$L^3 = (0.914)^3 = 0.764\ m^3$

Shaft alone:

$\omega_n = 9.87\sqrt{\frac{EI}{ML^3}} = 9.87\sqrt{\frac{207 \times 10^9 (1.024 \times 10^{-7})}{8.12\,(0.764)}}$

$= 9.87(58.4) = 577$ rad/s

$= 577 \frac{60}{2\pi} = 5510$ r/min

b) Disk A alone on a massless shaft:

$\omega_n = \sqrt{\frac{3EIL}{Ma^2b^2}}$

$= \sqrt{\frac{3(207 \times 10^9)(1.024 \times 10^{-7})\,0.914}{23\,(0.305)^2\,(0.609)^2}}$

$= \sqrt{73\,200} = 271$ rad/s

Disk B alone on a massless shaft:

$\omega_n = \sqrt{\frac{3EIL}{Ma^2b^2}}$

$= \sqrt{\frac{3(207 \times 10^9)(1.024 \times 10^{-7})\,0.914}{34\,(0.508)^2\,(0.406)^2}}$

$= \sqrt{40\,000} = 200$ rad/s

22-6 (CONT.)

Then

$\frac{1}{\omega_n^2} = \frac{1}{\omega_s^2} + \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}$

$\frac{1}{\omega_n^2} = \frac{1}{577^2} + \frac{1}{271^2} + \frac{1}{200^2}$

$\frac{1}{\omega_n^2} = 3.00 \times 10^{-6} + 13.6 \times 10^{-6} + 25 \times 10^{-6}$

$= 41.6 \times 10^{-6}$

$\omega_n^2 = \frac{1}{41.6 \times 10^{-6}} = 24\,038$

$\omega_n = 155$ rad/s $= 155 \frac{60}{2\pi} = 1480$ r/min

c) $\frac{1}{\omega_n^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}$

$= \frac{1}{271^2} + \frac{1}{200^2}$

$= 13.6 \times 10^{-6} + 25 \times 10^{-6} = 38.6 \times 10^{-6}$

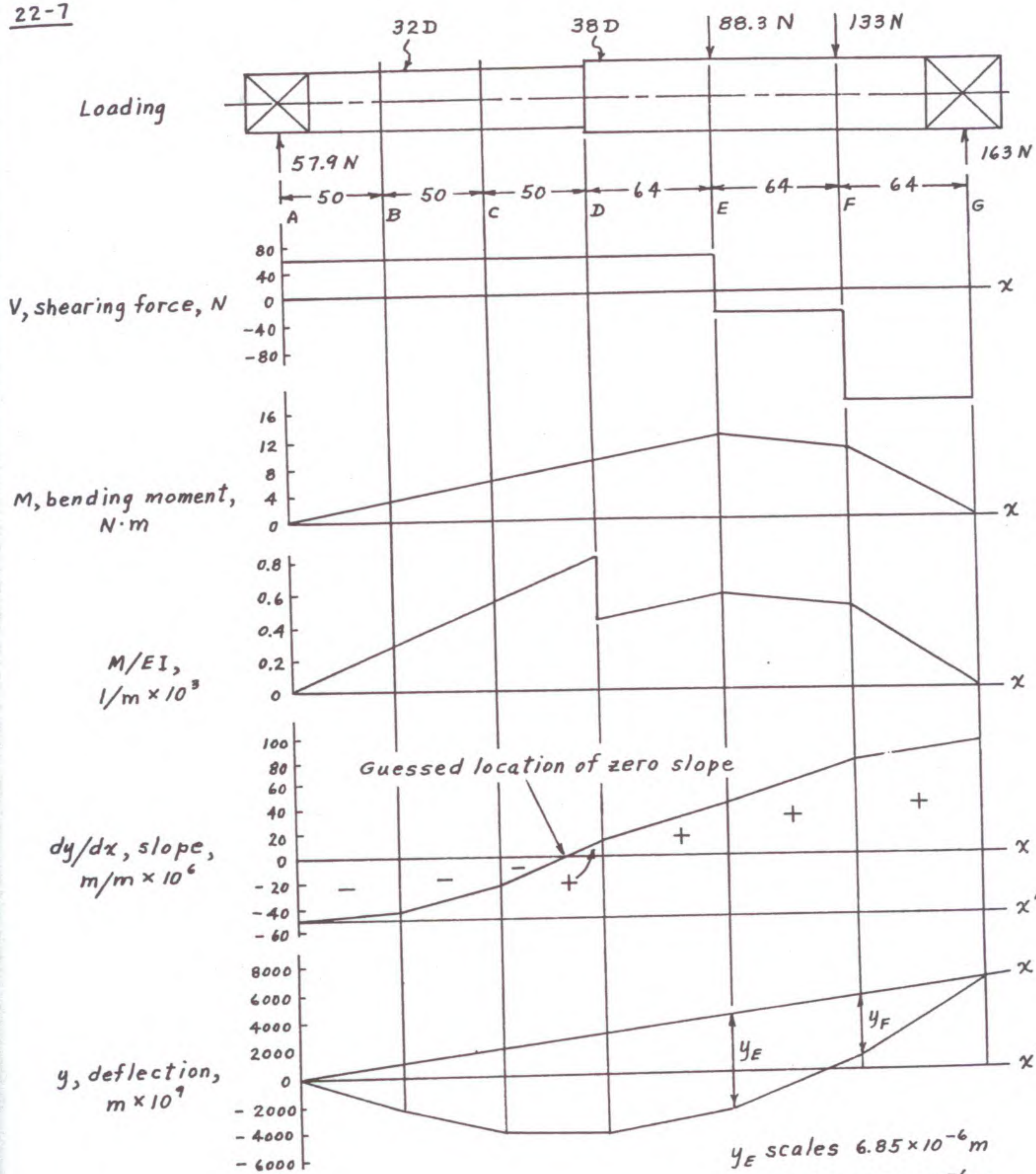
$\omega_n^2 = 25\,910$, $\omega_n = 161$ rad/s

$\omega_n = 161 \frac{60}{2\pi} = 1537$ r/min

d) Error $= \left(\frac{1537 - 1480}{1480}\right)$

$= 0.0385 = 3.85\,\%$

22-7



y_E scales 6.85×10^{-6} m

y_F scales 4.36×10^{-6} m

22-7 (CONT.)

Loads:

$9 \times 9.806 = 88.3\ N$

$13.6 \times 9.806 = 133\ N$

Bearing reactions:

$$R_A = \frac{(133 \times 64) + (88.3 \times 128)}{342} = 57.9\ N$$

$$R_G = \frac{(88.3 \times 214) + (133 \times 278)}{342} = 163\ N$$

Moments:

$M_B = 57.9\,(0.050) = 2.895\ N \cdot m$

$M_C = 57.9\,(0.100) = 5.790\ N \cdot m$

$M_D = 57.9\,(0.150) = 8.685\ N \cdot m$

$M_E = 57.9\,(0.214) = 12.391\ N \cdot m$

$M_F = 57.9\,(0.278) - 88.3\,(0.064)$
$= 10.445\ N \cdot m$

Moments of inertia:

$$I_1 = \frac{\pi\,(0.032)^4}{64} = 5.147 \times 10^{-8}\ m^4$$

$$I_2 = \frac{\pi\,(0.038)^4}{64} = 10.24 \times 10^{-8}\ m^4$$

M/EI values:

$$M_B/EI_1 = \frac{2.895}{207 \times 10^9 \times 5.147 \times 10^{-8}} = 0.272 \times 10^{-3}\ \frac{1}{m}$$

$$M_C/EI_1 = \frac{5.790}{207 \times 10^9 \times 5.147 \times 10^{-8}} = 0.544 \times 10^{-3}$$

$$M_D/EI_1 = \frac{8.685}{207 \times 10^9 \times 5.147 \times 10^{-8}} = 0.816 \times 10^{-3}$$

$$M_D/EI_2 = \frac{8.685}{207 \times 10^9 \times 10.24 \times 10^{-8}} = 0.416 \times 10^{-3}$$

$$M_E/EI_2 = \frac{12.391}{207 \times 10^9 \times 10.24 \times 10^{-8}} = 0.585 \times 10^{-3}$$

$$M_F/EI_2 = \frac{10.445}{207 \times 10^9 \times 10.24 \times 10^{-8}} = 0.493 \times 10^{-3}$$

Areas in M/EI diagram:

$$A \text{ to } B = \frac{1}{2}(0.050)\,0.272 \times 10^{-3} = 6.8 \times 10^{-6}\ \frac{m}{m}$$

$$B \text{ to } C = 0.050\left(\frac{0.272 + 0.544}{2}\right)10^{-3} = 20.4 \times 10^{-6}$$

$$C \text{ to } D = 0.050\left(\frac{0.544 + 0.816}{2}\right)10^{-3} = 34.0 \times 10^{-6}$$

$$D \text{ to } E = 0.064\left(\frac{0.410 + 0.585}{2}\right)10^{-3} = 31.8 \times 10^{-6}$$

$$E \text{ to } F = 0.064\left(\frac{0.585 + 0.493}{2}\right)10^{-3} = 34.5 \times 10^{-6}$$

$$F \text{ to } G = \frac{1}{2}(0.064)\,0.493 \times 10^{-3} = 15.8 \times 10^{-6}$$

Ordinates from x' axis in slope diagram:

$B = 6.8 \times 10^{-6}\ \frac{m}{m}$

$C = (6.8 + 20.4)\,10^{-6} = 27.2 \times 10^{-6}$

$D = (27.2 + 34.0)\,10^{-6} = 61.2 \times 10^{-6}$

$E = (61.2 + 31.8)\,10^{-6} = 93 \times 10^{-6}$

$F = (93 + 34.5)\,10^{-6} = 128 \times 10^{-6}$

$G = (128 + 15.8)\,10^{-6} = 143 \times 10^{-6}$

22-7(cont.)

Areas in slope diagram:

$$A \text{ to } B = -0.050(48\times10^{-6}) = -2400\times10^{-9}\,\text{m}$$

$$B \text{ to } C = -0.050(34\times10^{-6}) = -1700\times10^{-9}$$

$$C \text{ to } D = -0.033(11.6\times10^{-6}) + 0.017(5.8\times10^{-6}) = -284\times10^{-9}$$

$$D \text{ to } E = +0.064(27.2\times10^{-6}) = +1741\times10^{-9}$$

$$E \text{ to } F = +0.064(59.2\times10^{-6}) = +3789\times10^{-9}$$

$$F \text{ to } G = +0.064(84.4\times10^{-6}) = 5402\times10^{-9}$$

Ordinates from x' axis in deflection diagram:

$$B = -2400\times10^{-9}\,\text{m}$$

$$C = (-2400-1700)10^{-9} = -4100\times10^{-9}$$

$$D = (-4100-284)10^{-9} = -4384\times10^{-9}$$

$$E = (-4384+1741)10^{-9} = -2643\times10^{-9}$$

$$F = (-2643+3789)10^{-9} = +1146\times10^{-9}$$

$$G = (+1146+5402)10^{-9} = +6548\times10^{-9}$$

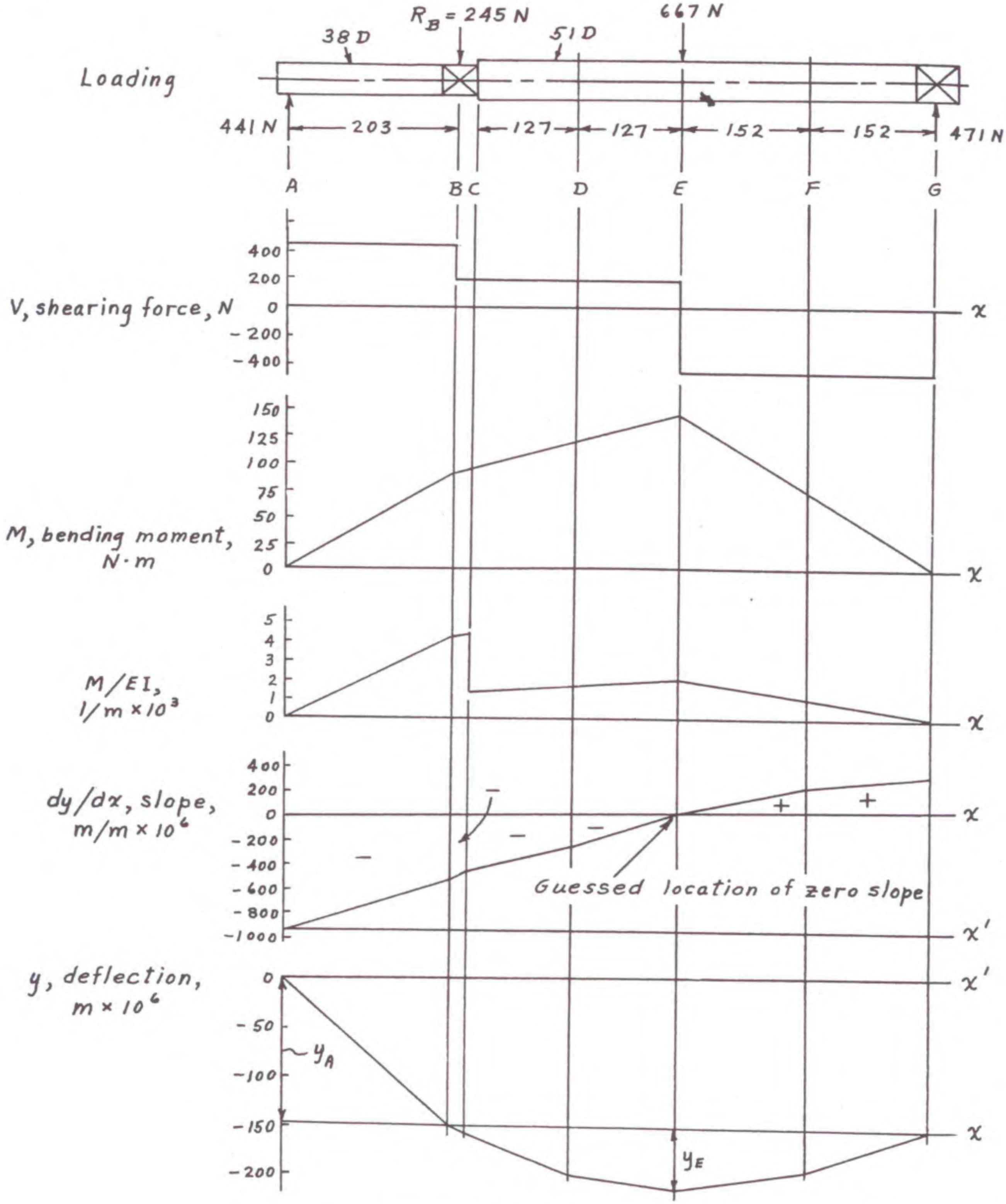
M, kg	y_{st}, m	y^2	My	My^2
9	6.85×10^{-6}	46.9×10^{-12}	61.7×10^{-6}	422×10^{-12}
13.6	4.36×10^{-6}	19.0×10^{-12}	59.3×10^{-6}	258×10^{-12}
		$\Sigma =$	121.0×10^{-6}	680×10^{-12}

$$\omega_n = \sqrt{\frac{g\Sigma My}{\Sigma My^2}} = \sqrt{\frac{9.806\times121.0\times10^{-6}}{680\times10^{-12}}}$$

$$= \sqrt{1.745\times10^{6}} = 1321\ \text{rad/s}$$

$$= 1321\,\frac{60}{2\pi} = \underline{\underline{12\,614\ \text{r/min}}}$$

22-8



y_A scales 148×10^{-6} m $\quad$ y_B scales 64×10^{-6} m

22-8 (CONT.)

Loads:

$45 \times 9.806 = 441\ N$

$68 \times 9.806 = 667\ N$

Bearing reactions:

$$R_B = \frac{(441 \times 780) - (667 \times 304)}{577} = 245\ N$$

$$R_G = \frac{(441 \times 203) + (667 \times 273)}{577} = 471\ N$$

Moments:

$M_B = 441 \times 0.203 = 89.523\ N \cdot m$

$M_C = (441 \times 0.222) - (245 \times 0.019) = 93.247$

$M_D = (441 \times 0.349) - (245 \times 0.146) = 118.139$

$M_E = (441 \times 0.476) - (245 \times 0.273) = 143.031$

$M_F = 471 \times 0.152 = 71.592$

Moments of inertia

$$I_1 = \frac{\pi (0.038)^4}{64} = 0.1024 \times 10^{-6}\ m^4$$

$$I_2 = \frac{\pi (0.051)^4}{64} = 0.3321 \times 10^{-6}\ m^4$$

M/EI values:

$$\frac{M_B}{EI_1} = \frac{89.523}{207 \times 10^9 \times 0.1024 \times 10^{-6}}$$

$$= 4.23 \times 10^{-3}\ \frac{1}{m}$$

$$\frac{M_C}{EI_1} = \frac{93.247}{207 \times 10^9 \times 0.1024 \times 10^{-6}} = 4.40 \times 10^{-3}$$

$$\frac{M_C}{EI_2} = \frac{93.247}{207 \times 10^9 \times 0.3321 \times 10^{-6}} = 1.36 \times 10^{-3}$$

$$\frac{M_D}{EI_2} = \frac{118.139}{207 \times 10^9 \times 0.3321 \times 10^{-6}} = 1.72 \times 10^{-3}$$

$$\frac{M_E}{EI_2} = \frac{143.031}{207 \times 10^9 \times 0.3321 \times 10^{-6}} = 2.08 \times 10^{-3}$$

$$\frac{M_F}{EI_2} = \frac{71.592}{207 \times 10^9 \times 0.3321 \times 10^{-6}} = 1.04 \times 10^{-3}$$

Areas in M/EI diagram:

$A \text{ to } B = \frac{1}{2}(0.203)\, 4.23 \times 10^{-3} = 429 \times 10^{-6}\ \frac{m}{m}$

$B \text{ to } C = 0.019 \left(\frac{4.23 + 4.40}{2}\right) 10^{-3} = 82.0 \times 10^{-6}$

$C \text{ to } D = 0.127 \left(\frac{1.36 + 1.72}{2}\right) 10^{-3} = 196 \times 10^{-6}$

$D \text{ to } E = 0.127 \left(\frac{1.72 + 2.08}{2}\right) 10^{-3} = 241 \times 10^{-6}$

$E \text{ to } F = 0.152 \left(\frac{2.08 + 1.04}{2}\right) 10^{-3} = 237 \times 10^{-6}$

$F \text{ to } G = \frac{1}{2}(0.152)\, 1.04 \times 10^{-3} = 79.0 \times 10^{-6}$

Ordinates from x' axis in slope diagram:

$B = 429 \times 10^{-6}\ m/m$

$C = (429 + 82)\, 10^{-6} = 511 \times 10^{-6}$

$D = (511 + 196)\, 10^{-6} = 707 \times 10^{-6}$

$E = (707 + 241)\, 10^{-6} = 948 \times 10^{-6}$

$F = (948 + 237)\, 10^{-6} = 1185 \times 10^{-6}$

$G = (1185 + 79.0)\, 10^{-6} = 1264 \times 10^{-6}$

Areas in slope diagram:

$A \text{ to } B = -0.203\,(733.5 \times 10^{-6}) = -148.9 \times 10^{-6}\ m$

$B \text{ to } C = -0.019\,(478 \times 10^{-6}) = -9.082 \times 10^{-6}$

$C \text{ to } D = -0.127\,(339 \times 10^{-6}) = -43.05 \times 10^{-6}$

$D \text{ to } E = -\frac{1}{2}(0.127)\, 241 \times 10^{-6} = -15.30 \times 10^{-6}$

$E \text{ to } F = \frac{1}{2}(0.152)\, 237 \times 10^{-6} = 18.01 \times 10^{-6}$

$F \text{ to } G = 0.152\,(276.5 \times 10^{-6}) = 42.03 \times 10^{-6}$

Ordinates from x' axis in deflection diagram:

$B = -148.9 \times 10^{-6}\ m$

$C = (-148.9 - 9.082)\, 10^{-6} = -158 \times 10^{-6}$

$D = (-158 - 43.05)\, 10^{-6} = -201 \times 10^{-6}$

$E = (-201 - 15.3)\, 10^{-6} = -216 \times 10^{-6}$

$F = (-216 + 18.01)\, 10^{-6} = -198 \times 10^{-6}$

$G = (-198 + 42.03)\, 10^{-6} = -156 \times 10^{-6}$

22-8 (CONT.)

M, kg	y_{st}, m	y^2	My	My^2
45	1.48×10^{-4}	2.19×10^{-8}	66.6×10^{-4}	98.55×10^{-8}
68	0.64×10^{-4}	0.410×10^{-8}	43.52×10^{-4}	27.88×10^{-8}
		$\Sigma=$	110.12×10^{-4}	126.43×10^{-8}

$$\omega_n = \sqrt{\frac{g\Sigma My}{\Sigma My^2}} = \sqrt{\frac{9.806\times110.12\times10^{-4}}{126.43\times10^{-8}}}$$

$$=\sqrt{85410} = 292 \text{ rad/s}$$

$$= 292\left(\frac{60}{2\pi}\right) = \underline{\underline{2790 \text{ r/min}}}$$

22-9

$\omega_n = 1$ cycle/sec $= 2\pi$ rad/sec

$$\text{Gravity force} = 0.283\,\frac{\pi(4.75)^2 1.20}{4}$$

$$= 6.018 \text{ lbf}$$

$$M = \frac{\text{gravity force}}{g} = \frac{6.018}{32.2\times12}$$

$$= 0.0156\ \frac{\text{lbf}\cdot\text{sec}^2}{\text{in}}$$

$$= Mr^2 = M\left(\frac{R}{\sqrt{2}}\right)^2 = \frac{MR^2}{2}$$

$$= 0.0156\,\frac{(2.375)^2}{2} = 0.0440 \text{ lbf-in-sec}^2$$

$$\omega_n = \sqrt{\frac{\pi d^4 G}{32 I l}}, \quad d^4 = \frac{32\omega_n^2 I l}{\pi G}$$

$$d^4 = \frac{32(2\pi)^2\, 0.0440\times24}{\pi\times11.5\times10^6}$$

$$= 36.925\times10^{-6}, \quad \underline{\underline{d = 0.0780 \text{ in}}}$$

22-10

a) $$J = \frac{\pi d^4}{32} = \frac{\pi(4)^2}{32} = 25.132 \text{ in}^4$$

$$k_t = \frac{JG}{l} = \frac{25.132\times11.5\times10^6}{32}$$

$$= 9.032\times10^6 \text{ in-lb/rad}$$

22-10 (CONT.)

Generator : $I_1 = 1600$ lbf-sec-in

Motor : $I_2 = 2000$ lbf-sec-in

$$\omega_n = \sqrt{k_t\frac{I_1+I_2}{I_1 I_2}}$$

$$= \sqrt{9.032\times10^6\,\frac{1600+2000}{1600(2000)}}$$

$$\sqrt{10161} = 100.1 \text{ rad/sec}$$

$$\omega_n = \frac{100.1}{2\pi} = \underline{\underline{16.04 \text{ cycles/sec}}}$$

b) $$l_1 = \frac{I_2 l}{I_1+I_2} = \frac{2000(32)}{1600+2000} = 17.78 \text{ in}$$

$$l_2 = l - l_1 = 32 - 17.78 = 14.22 \text{ in}$$

Thus node is 14.22 in from motor

22-11

a) $$J = \frac{\pi d^4}{32} = \frac{\pi(0.065)^4}{32} = 1.752\times10^{-6}\text{ m}^4$$

$$k_t = \frac{JG}{l} = \frac{1.752\times10^{-6}(79.3\times10^9)}{0.308}$$

$$= 4.511\times10^5 \text{ N}\cdot\text{m/rad}$$

Generator:

$$I_1 = Mr^2 = 7.20(0.109)^2 = 0.08554 \text{ kg}\cdot\text{m}^2$$

Turbine:

$$I_2 = Mr^2 = 14.2(0.232)^2 = 0.7643 \text{ kg}\cdot\text{m}^2$$

$$\omega_n = \sqrt{k_t\frac{I_1+I_2}{I_1 I_2}}$$

$$= \sqrt{4.511\times10^5\,\frac{0.08554+0.7643}{(0.08554)(0.7643)}}$$

$$= \sqrt{5.864\times10^6} = 2422 \text{ rad/s}$$

$$= \frac{2422(60)}{2\pi} = \underline{\underline{23128 \text{ cycles/min}}}$$

22-11 (CONT.)

b) $$l_1 = \frac{I_2 l}{I_1 + I_2} = \frac{0.7643(0.308)}{0.08554 + 0.7643}$$

$$= 0.277\ m = 277\ mm$$

$$l_2 = l - l_1 = 308 - 277 = 31\ mm$$

Thus node is 31 mm from turbine

c) For resonance $Nn_c = f_n$

$$15 n_c = 23128$$

$$n_c = 1542\ r/min$$

22-12

$$J_1 = \frac{\pi d_1^4}{32} = \frac{\pi(0.045)^4}{32} = 40.25 \times 10^{-8}\ m^4$$

$$J_2 = \frac{\pi d_2^4}{32} = \frac{\pi(0.028)^4}{32} = 6.03 \times 10^{-8}\ m^4$$

$$k_1 = \frac{J_1 G}{l_1} = \frac{40.25 \times 10^{-8}(79.3 \times 10^9)}{0.350}$$

$$= 91\ 200\ N \cdot m/rad$$

$$k_2 = \frac{J_2 G}{l_2} = \frac{6.03 \times 10^{-8}(79.3 \times 10^9)}{0.250}$$

$$= 19130\ N \cdot m/rad$$

$$\frac{1}{k_t} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$= \frac{1}{91\ 200} + \frac{1}{19130}$$

$$= 1.096 \times 10^{-5} + 5.227 \times 10^{-5}$$

$$= 6.323 \times 10^{-5}$$

$$k_t = \frac{1}{6.323 \times 10^{-5}} = 15\ 820\ N \cdot m/rad$$

$$\omega_n = \sqrt{k_t \frac{I_1 + I_2}{I_1 I_2}}$$

$$= \sqrt{15\ 820 \frac{0.878 + 0.439}{(0.878)(0.439)}}$$

$$= \sqrt{54\ 054} = 232\ rad/s$$

$$= \frac{232}{2\pi} = 36.9\ Hz$$

22-13

Equivalent spring constant

$$k_t = \frac{1}{(1/k_1) + (1/n^2 k_2)}$$

$$= \frac{1}{[1/5.00 \times 10^8] + [1/(4 \times 1.92 \times 10^8)]}$$

$$= \frac{1}{0.200 \times 10^{-8} + 0.1302 \times 10^{-8}}$$

$$= 3.028 \times 10^8\ N \cdot m/rad$$

$$\omega_n = \sqrt{k_t \frac{I_1 + n^2 I_2}{I_1 n^2 I_2}}$$

$$= \sqrt{3.028 \times 10^8 \frac{5.20 + 4(0.260)}{5.20(4)\ 0.260}}$$

$$= \sqrt{3.494 \times 10^8} = 18\ 690\ rad/s$$

$$= \frac{18\ 690(60)}{2\pi} = 179\ 000\ \text{cycles/min}$$

22-14

Speed ratio, $n = 32/18 = 1.778$

$$n^2 = 3.161$$

Equivalent spring constant

$$k_t = \frac{1}{(1/k_1) + (1/n^2 k_2)}$$

$$= \frac{1}{[1/1.30 \times 10^8] + [1/(3.161)\ 0.325 \times 10^8]}$$

$$= \frac{1}{0.7692 \times 10^{-8} + 0.9734 \times 10^{-8}}$$

$$= \frac{1}{1.7426 \times 10^{-8}} = 0.5739 \times 10^8\ N \cdot m/rad$$

22-14 (CONT.)

$$\omega_n = \sqrt{k_t \frac{I_1 + n^2 I_2}{I_1 n^2 I_2}}$$

$$= \sqrt{0.5739 \times 10^8 \left[\frac{1.70 + (3.161)\,0.93}{1.70\,(3.161)\,0.93}\right]}$$

$$= \sqrt{0.5739 \times 10^8 \,\frac{4.64}{4.998}} = 7299 \text{ rad/s}$$

$$= \frac{7299\,(60)}{2\pi} = \underline{\underline{69\,700 \text{ cycles/min}}}$$